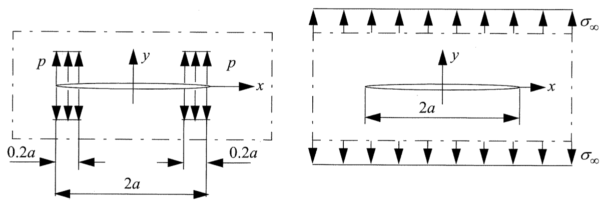
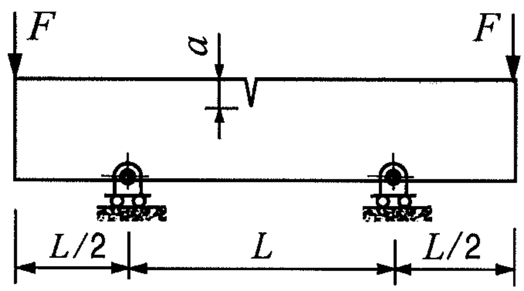
# RAK-33060 Fracture mechanics and fatigue

## 2. Home exercise - weight functions, elastic-plastic FM, fatigue, etc.

1. An internal crack in a large plate is loaded with an internal pressure $p$ along 10% of its length adjacent to the crack tips. How large should this pressure to be to give the same contribution to the stress intensity factor as an outer nominal tensile traction $\sigma_\infty$?



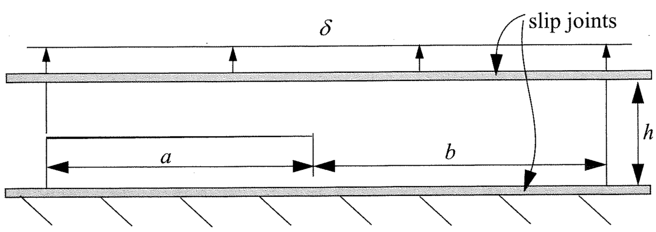

2. Determine the maximum allowed load for the cracked beam shown below. The length of the crack is $a = 5$ mm. Use the elastic-plastic correction by (a) Irwin or (b) the Dugdale's yield strip model. Dimensions are: length $L = 1$ m, height $h = 50$ mm, width $b = 5$ mm, Young's modulus $E = 210$ GPa, fracture toughness $K_{\mathrm{Ic}} = 75$ MPam$^{1/2}$ and the yield stress $\sigma_0 = 340$ MPa.



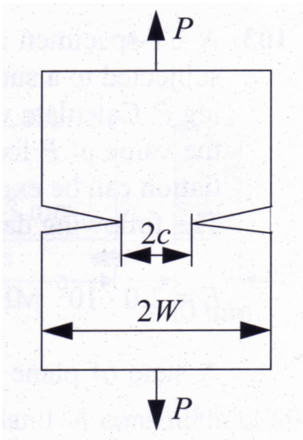

3. The horizontal boundaries of a very long plate, shown below ($a \gg h, b \gg h$), are fastened to a foundation via slip joints and are subjected to a prescribed displacement $\delta$. The material is elasto-plastic ($E = 200$ GPa, $\sigma_0 = 250$ MPa) with a uniaxial stress-strain behaviour

$$\sigma = E\varepsilon, \quad \text{if} \quad \varepsilon < \varepsilon_0, \quad \text{and} \quad \sigma = E\varepsilon_0(\varepsilon/\varepsilon_0)^n \quad \text{if} \quad \varepsilon \geq \varepsilon_0,$$

where $\varepsilon_0 = \sigma_0/E$ and $n = 0.1$. Determine the displacement $\delta$ at initiation of crack growth if $h = 100$ mm and the fracture toughness is $J_{\mathrm{Ic}} = 120\mathrm{kNm/m^2}$. Assume plane stress contitions and the crack growth criteria is $J = J_{\mathrm{Ic}}$.



4. A long, very deeply cracked plate shown in the figure below is subjected to a tensile load $P = 4$ MN. Use the R6 method to estimate the minimun size of the ligament for which no risk of crack growth exists. Use equation (9.68a) on page 416 in Anderson's book for the $K_\mathrm{r}$. Plane deformation can be assumed. The stress intensity factor for this case can be written as
$$K_\mathrm{I} = \frac{P}{t\sqrt{\pi c}},$$
where $t$ is the thickness of the plate. Apply the data: yield stress $\sigma_\mathrm{y} = 400$ MPa, fracture toughness $K_\mathrm{Ic} = 150$ MPa$\sqrt{\mathrm{m}}$, plate thickness is $t = 4$ cm.




See the figure on the last page for the limit load.

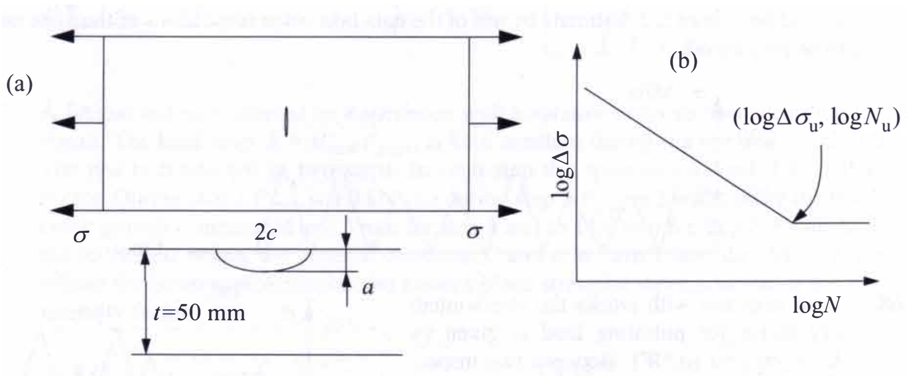
5. The Basquin relation $\sigma_\mathrm{af} = \sigma'_\mathrm{f}(2N)^{-b}$ and the S-N curve describe the same phenomenon. Assume that the S-N curve is linear in the double logarithmic scale between $10^3 \leq N \leq 10^6$ and the fatigue strengths at $N = 10^3$ is $\xi_3\sigma_\mathrm{u}$ and at $N = 10^6$ is $\xi_6\sigma_\mathrm{u}$. How the parameters in both descriptions are related? Consider as an example a high-strength steel for which $\sigma_\mathrm{u} = 1500$ MPa, $\xi_3 = 0.9$ and $\xi_6 = 0.5$.

6. Construct the S-N curve ($R = -1$) of a plate with a surface crack and made of high strength steel $\sigma_y = 1100$ MPa, $K_{Ic} = 60$ MPa$\sqrt{\text{m}}$. During manufacturing inspection all cracks deeper than 1 mm are detected. It can be assumed that the crack grows in a self similar fashion ($c = 1.25a$) and can be described by the Paris law

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\left(\frac{\Delta K_{\mathrm{I}}}{K_{\mathrm{ref}}}\right)^n.$$

The constants have the values $C = 10^{-7}$ m/cycle, $K_{\mathrm{ref}} = 20$ MPa$\sqrt{\text{m}}$, $n = 3.2$ and the threshold value $\Delta K_{\mathrm{th}} = 8$ MPa$\sqrt{\text{m}}$. The maximum stress can occasionally reach the level 375 MPa.

**Hint**. Use the threshold value to compute the kink point $(\Delta\sigma_u, N_u)$ in the figure b below. For the stress intensity factor see page 4.




7. In the multiaxial Findley fatigue criterion the fatigue takes place in a plane where the expression $\tau_{a,n} + k\sigma_n$ attains its maximum, i.e.

$$\max(\tau_{a,n} + k\sigma_n) = f, \tag{1}$$

where $k$ and $f$ are material parameterd which can be determined from two tests. Determine $k$ and $f$ if we know the fatigue limit for fully reversed uniaxial normal $\sigma_{-1} = \sigma_{a,R=-1}$ and shear $\tau_{-1} = \tau_{a,R=-1}$. Determine also the critical angles for both loading cases.

$$K_{1max} = \sigma_0\sqrt{\pi a}\ \ f_7\left(\frac{a}{c}, \frac{a}{B}\right)$$

$$f_7\left(\frac{a}{c}, \frac{a}{t}\right) = Q^{-1/2}\left[M_1 + M_2\left(\frac{a}{B}\right)^2 + M_3\left(\frac{a}{B}\right)^4\right]$$

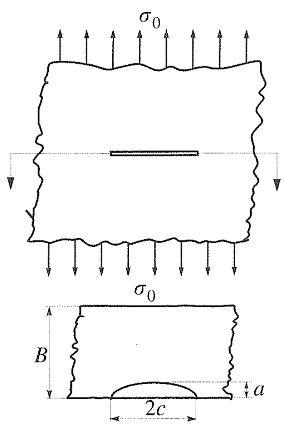
$$M_1 = 1.13 - 0.09\left(\frac{a}{c}\right)$$

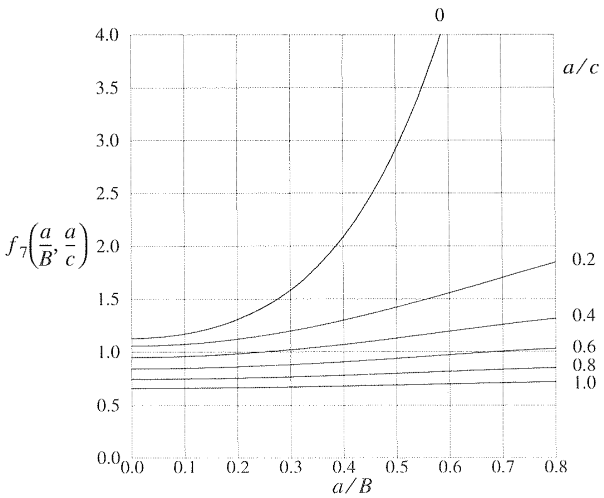
$$M_2 = -0.54 + 0.89/(0.2 + a/c)$$

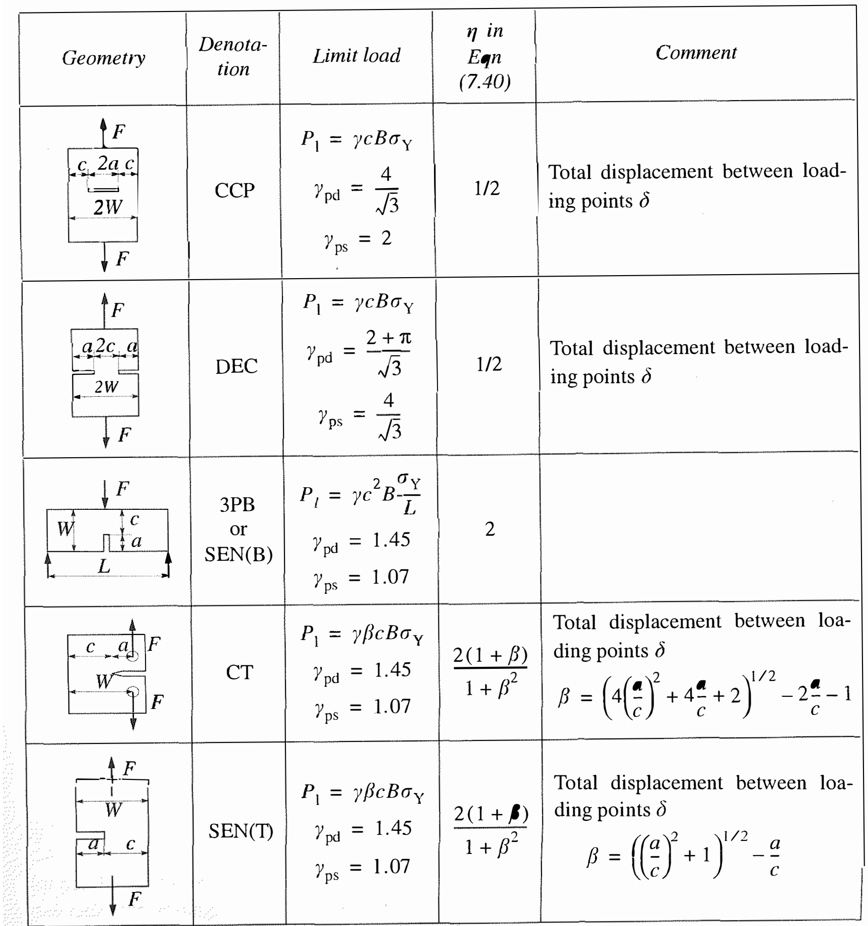
$$M_3 = 0.5 - 1.0/(0,65 + a/c) + 14(1 - a/c)^{24}$$

$$Q = 1 + 1.464(a/c)^{1,65}$$

Ref. Newman and Raju [64]





| *Geometry* | *Denotation* | *Limit load* | *$\eta$ in Eqn (7.40)* | *Comment* |
|---|---|---|---|---|
| | CCP | $P_l = \gamma c B \sigma_Y$<br>$\gamma_{pd} = \frac{4}{\sqrt{3}}$<br>$\gamma_{ps} = 2$ | 1/2 | Total displacement between loading points $\delta$ |
| | DEC | $P_l = \gamma c B \sigma_Y$<br>$\gamma_{pd} = \frac{2+\pi}{\sqrt{3}}$<br>$\gamma_{ps} = \frac{4}{\sqrt{3}}$ | 1/2 | Total displacement between loading points $\delta$ |
| | 3PB or SEN(B) | $P_l = \gamma c^2 B \frac{\sigma_Y}{L}$<br>$\gamma_{pd} = 1.45$<br>$\gamma_{ps} = 1.07$ | 2 | |
| | CT | $P_l = \gamma \beta c B \sigma_Y$<br>$\gamma_{pd} = 1.45$<br>$\gamma_{ps} = 1.07$ | $\frac{2(1+\beta)}{1+\beta^2}$ | Total displacement between loading points $\delta$<br>$\beta = \left(4\left(\frac{a}{c}\right)^2 + 4\frac{a}{c} + 2\right)^{1/2} - 2\frac{a}{c} - 1$ |
| | SEN(T) | $P_l = \gamma \beta c B \sigma_Y$<br>$\gamma_{pd} = 1.45$<br>$\gamma_{ps} = 1.07$ | $\frac{2(1+\beta)}{1+\beta^2}$ | Total displacement between loading points $\delta$<br>$\beta = \left(\left(\frac{a}{c}\right)^2 + 1\right)^{1/2} - \frac{a}{c}$ |

Fig. 29. Limit loads for some selected geometries. Index *pd* denotes plane deformation while *ps* denotes plane stress.

Figures from F. Nilsson, *Fracture Mechanics - from Theory to Applications*, 2001, on page 181 and 71. Published by the Department of Solid Mechanics, Royal Institute of Technology, Stockholm, Sweden.