RAK-33060 Fracture mechanics and fatigue

2. Home exercise - weight functions, elastic-plastic FM, fatigue, etc.

1. An internal crack in a large plate is loaded with an internal pressure $p$ along 10\% of its length adjacent to the crack tips. How large should this pressure to be to give the same contribution to the stress intensity factor as an outer nominal tensile traction $\sigma_\infty$?

2. Determine the maximum allowed load for the cracked beam shown below. The length of the crack is $a = 5$ mm. Use the elastic-plastic correction by (a) Irwin or (b) the Dugdale’s yield strip model. Dimensions are: length $L = 1$ m, height $h = 50$ mm, width $b = 5$ mm, Young’s modulus $E = 210$ GPa, fracture toughness $K_{Ic} = 75$ MPam$^{1/2}$ and the yield stress $\sigma_0 = 340$ MPa.

3. The horizontal boundaries of a very long plate, shown below ($a \gg h, b \gg h$), are fastened to a foundation via slip joints and are subjected to a prescribed displacement $\delta$. The material is elasto-plastic ($E = 200$ GPa, $\sigma_0 = 250$ MPa) with a uniaxial stress-strain behaviour

$$\sigma = E\varepsilon, \quad \text{if} \quad \varepsilon < \varepsilon_0, \quad \text{and} \quad \sigma = E\varepsilon_0(\varepsilon/\varepsilon_0)^n \quad \text{if} \quad \varepsilon \geq \varepsilon_0,$$

where $\varepsilon_0 = \sigma_0/E$ and $n = 0.1$. Determine the displacement $\delta$ at initiation of crack growth if $h = 100$ mm and the fracture toughness is $J_{Ic} = 120$kNm/m$^2$. Assume plane stress conditions and the crack growth criteria is $J = J_{Ic}$.
4. A long, very deeply cracked plate shown in the figure below is subjected to a tensile load $P = 4$ MN. Use the R6 method to estimate the minimum size of the ligament for which no risk of crack growth exists. Use equation (9.68a) on page 416 in Anderson’s book for the $K_r$. Plane deformation can be assumed. The stress intensity factor for this case can be written as

$$K_I = \frac{P}{t\sqrt{\pi c}},$$

where $t$ is the thickness of the plate. Apply the data: yield stress $\sigma_y = 400$ MPa, fracture toughness $K_{Ic} = 150$ MPa$\sqrt{m}$, plate thickness is $t = 4$ cm.

See the figure on the last page for the limit load.

5. The Basquin relation $\sigma_{af} = \sigma_f'(2N)^{-b}$ and the S-N curve describe the same phenomenon. Assume that the S-N curve is linear in the double logarithmic scale between $10^3 \leq N \leq 10^6$ and the fatigue strengths at $N = 10^3$ is $\xi_3 \sigma_u$ and at $N = 10^6$ is $\xi_6 \sigma_u$. How the parameters in both descriptions are related? Consider as an example a high-strength steel for which $\sigma_u = 1500$ MPa, $\xi_3 = 0.9$ and $\xi_6 = 0.5$. 
6. Construct the S-N curve \((R = -1)\) of a plate with a surface crack and made of high strength steel \(\sigma_y = 1100\) MPa, \(K_{ic} = 60\) MPa\(\sqrt{m}\). During manufacturing inspection all cracks deeper than 1 mm are detected. It can be assumed that the crack grows in a self similar fashion \((c = 1.25a)\) and can be described by the Paris law

\[
\frac{da}{dN} = C \left( \frac{\Delta K_1}{K_{ref}} \right)^n.
\]

The constants have the values \(C = 10^{-7}\) m/cycle, \(K_{ref} = 20\) MPa\(\sqrt{m}\), \(n = 3.2\) and the threshold value \(\Delta K_{th} = 8\) MPa\(\sqrt{m}\). The maximum stress can occasionally reach the level 375 MPa.

**Hint.** Use the threshold value to compute the kink point \((\Delta \sigma_u, N_u)\) in the figure b below. For the stress intensity factor see page 4.

![Diagram](image)

7. In the multiaxial Findley fatigue criterion the fatigue takes place in a plane where the expression \(\tau_{a,n} + k\sigma_n\) attains its maximum, i.e.

\[
\max(\tau_{a,n} + k\sigma_n) = f,
\]

where \(k\) and \(f\) are material parameters which can be determined from two tests. Determine \(k\) and \(f\) if we know the fatigue limit for fully reversed uniaxial normal \(\sigma_{-1} = \sigma_{a,R=-1}\) and shear \(\tau_{-1} = \tau_{a,R=-1}\). Determine also the critical angles for both loading cases.
\[ K_{\text{max}} = \sigma_0 \sqrt{\pi a} \cdot f_\gamma \left( \frac{a}{B} \right) \]

\[ f_\gamma \left( \frac{a}{B} \right) = Q^{-1/2} \left[ M_1 + M_2 \left( \frac{a}{B} \right) + M_3 \left( \frac{a}{B} \right)^3 \right] \]

\[ M_1 = 1.13 - 0.09 \left( \frac{a}{c} \right) \]

\[ M_2 = -0.54 + 0.89/(0.2 + a/c) \]

\[ M_3 = 0.5 - 1.0/(0.65 + a/c) + 14(1 - a/c)^{3/4} \]

\[ Q = 1 + 1.464(a/c)^{1.65} \]

Ref. Newman and Raju [64]
The following approach, called the R6 method, has extensively and has been developed into a standard document, Milne et al. [61]. Basically, it is regarded as a correction for plastic deformation to the linear elastic fracture mechanics. The form of the correction is given by

$$J_f = J_c/UR6(F/P1)$$

(7.28)

This form can be written for any specific geometry, and strictly speaking, the correction will be different for different geometries and crack sizes. By numerical experiments, it has been found that a correction function of the form in (7.28) is insensitive to geometry and the following expression is a conservative estimate for practical interest.

$$J_f \leq \left( 1 - 0.14L_r \right)^{0.3} + 0.7e^{-0.65L_r^{1/2}}$$

(7.29)

This approximation is illustrated by the initiation criterion of Fig. 26. The upper bound shown is an upper limit which is in accordance with the procedure that neither linearly elastic fracture nor plastic collapse should occur. Diagrams of the type shown in Fig. 28 are often called failure assessment diagrams (FAD).

In both of the methods discussed, the estimation procedure is based on the linearly elastic stress-intensity factor and the limit load of the cracked structure. Some results for common geometries are shown in Fig. 29.

Fig. 29. Limit loads for some selected geometries. Index pd denotes plane deformation while ps denotes plane stress.