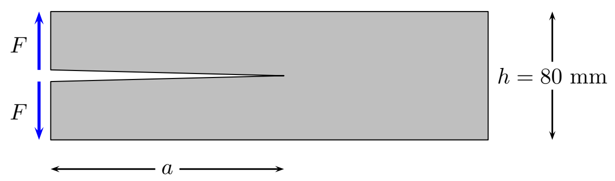
RAK-33060 Fracture mechanics and fatigue

6. Exercise

Problem 1. A DCB-specimen shown in the figure below is loaded by the force $P = 50$ kN. For the material $J_{\mathrm{Ic}} = 100$ kN/m and the yield strength is $\sigma_{\mathrm{y}} = 400$ MPa. Compute the critical crack length a_{cr} using the model of (a) Irvin and (b) Dugdale. The Young's modulus of the specimen is $E = 200$ GPa and the Poisson's ratio is $\nu = 0.3$. Thickness of the specimen is 50 mm.



Solution. First calculate the critical crack length based on the linear fracture mechanics. So the compliance function is:

$$\mathcal{G} = \frac{P^2}{2B}\frac{\mathrm{d}C}{\mathrm{dc}}.$$

Consider the opening of crack by symbol Δ, and the displacement of the beam head is:

$$\frac{\Delta}{2} = \frac{Pa^3}{3EI}.$$

Where I is the moment of inertia; $I = \frac{1}{12}B(\frac{h}{2})^3$. So the compliance function will be:

$$C = \frac{\Delta}{P} = \frac{64}{EB}(\frac{a}{h})^3.$$

The critical crack length occurs when $\mathcal{G}_{cr} = J_{Ic}$.

$$a_{\mathrm{cr}} = \frac{Bh}{P}(\frac{\mathcal{G}_{\mathrm{cr}}Eh}{4\sqrt{6}}) = 326\,\mathrm{mm}.$$

According to Irwin's model, the plasticity of the tip can be considered as an elastic problem by using an effective crack length

$$a_{\mathrm{eff}} = a + \frac{1}{2\pi}(\frac{K_{\mathrm{I}}}{\sigma_{\mathrm{y}}})^2.$$

The elastic solution gave a critical solution length of 326mm, so according to the Irwin's model $a_{eff} = 326$ mm. The critical crack length according to the Irwin's model is:

$$a_{\mathrm{cr}} = a_{\mathrm{eff}} + \frac{1}{2\pi}(\frac{K_{\mathrm{I}}}{\sigma_{\mathrm{y}}})^2 = a_{\mathrm{eff}} + \frac{1}{2\pi}\frac{J_{\mathrm{IC}}E}{\sigma_{\mathrm{y}}^2} = 324\,\mathrm{mm}.$$

II) Dugdale model:

$$J = \frac{8\sigma_{\mathrm{y}}^2}{\pi E}\ln(\frac{1}{\cos(\frac{\pi\sigma}{2\sigma_{\mathrm{y}}})}).$$

So now:

$$\sigma = \frac{K_{\mathrm{IC}}}{\sqrt{\pi a}} = \sqrt{\frac{JE}{\pi a}} \qquad \textit{and} \qquad J = \mathcal{G} = 96\frac{P^2a^2}{EB^2h^3}.$$

Thus, the critical crack length is solved from equation:

$$J_{\mathrm{IC}} + \frac{8\sigma_{\mathrm{y}}^2}{\pi E} a \ln(\cos(\frac{2\sqrt{6\pi}P\sqrt{a}}{\sigma_{\mathrm{y}} B h \sqrt{h}})) = 0.$$

With a numerical solution and converting the equation dimensionless:

$$\frac{\pi J_{\mathrm{IC}} E}{8\sigma_{\mathrm{y}}^2 a} \ln(\cos(\frac{2\sqrt{6\pi}P\sqrt{a}}{\sigma_{\mathrm{y}} B h \sqrt{h}})) = 0.$$

Using the Newton iteration the result is $a = 318$ mm.

Problem 2. A large steel plate of an elastic, perfectly plastic material with yield strength of $\sigma_{\mathrm{y}} = 500$ MPa contains a through-the-thickness crack with a length of 50 mm which is oriented perpendicularly to the uniform remotely applied tensile stress σ_∞. During increased loading the crack starts to grow when $\sigma_\infty = 300$ MPa. The Dugdale model can be assumed to be applicable to the present problem.

1. At which stress should a 150 mm long crack start to grow if initiation of crowth occurs when the crack surface opening at the rear end of the plastic zone reaches a critical level?
2. Calculate the size of the plastic zones at initiation of growth for the two initial crack lengths.

Solution. The crack tip opening displacement is:

$$\delta = a\frac{8\sigma_{\mathrm{y}}}{\pi E} \ln(\frac{1}{\cos(\frac{\pi}{2}\frac{\sigma_\infty}{\sigma_{\mathrm{y}}})})$$

Crack growth initiation when $\delta = \delta_{\mathrm{c}}$. Now:

$$a_1\frac{8\sigma_{\mathrm{y}}}{\pi E} \ln(\frac{1}{\cos(\frac{\pi}{2}\frac{\sigma_{1\infty}}{\sigma_{\mathrm{y}}})}) = a_2\frac{8\sigma_{\mathrm{y}}}{\pi E} \ln(\frac{1}{\cos(\frac{\pi}{2}\frac{\sigma_{2\infty}}{\sigma_{\mathrm{y}}})})$$

When $a_1 = 50$ mm and $\sigma_{1\infty} = 300$ MPa, and $a_2 = 150$ mm. So $\sigma_{2\infty} =$?

$$a_1 \ln(\frac{1}{\cos(\frac{\pi}{2}\frac{\sigma_{1\infty}}{\sigma_{\mathrm{y}}})}) = a_2 \ln(\frac{1}{\cos(\frac{\pi}{2}\frac{\sigma_{2\infty}}{\sigma_{\mathrm{y}}})})$$

$$(\cos(\frac{\pi}{2}\frac{\sigma_{1\infty}}{\sigma_{\mathrm{y}}}))^{a_1} = (\cos(\frac{\pi}{2}\frac{\sigma_{2\infty}}{\sigma_{\mathrm{y}}}))^{a_2}$$

So:

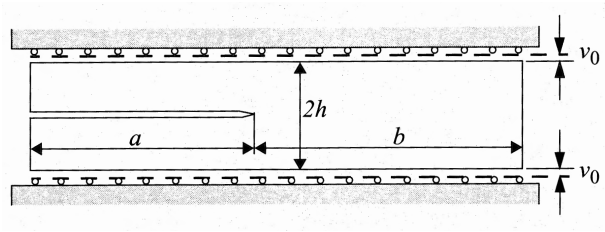
$$\sigma_{2\infty} = \frac{2\sigma_y}{\pi} \arccos((cos(\frac{\pi}{2}\frac{\sigma_{1\infty}}{\sigma_y}))^{\frac{a_1}{a_2}}) = 184\,\mathrm{MPa}.$$

Part b) Size of the plastic zone is:

$$L_1 = a_1[\frac{1}{\cos(\frac{\pi}{2}\frac{\sigma_{1\infty}}{\sigma_{\mathrm{y}}})} - 1] = 35\,\mathrm{mm},$$

$$L_2 = a_2[\frac{1}{\cos(\frac{\pi}{2}\frac{\sigma_{2\infty}}{\sigma_{\mathrm{y}}})} - 1] = 29.1\,\mathrm{mm}.$$

Problem 3. A long strip ($h \ll a, h \ll b$) is subjected to constant vertical displacements $\pm v_0$ along the horizontal boundaries. The boundaries are supported so that no shear stress is transferred. The crack tip region can ne modelled by the Dugdale model, i.e. a constant stress σ_y is assumed to act between the cracl surfaces in the cohesive region. Fracture is assumed to occur when the crack opening of the rear end of the zone reaches the critical value δ_c. Calculate with aid of the J-integral the value v_0 at which fracture occurs. A plane stress state is assumed. The elastic constants are E and ν.



Solution. On paths Γ_1,Γ_2,Γ_4,Γ_5, $J = 0$.

$$J = \int_\Gamma (w dy - T_i \frac{\partial u_i}{\partial x} \, \mathrm{d}s) = J_3 = \int_{\Gamma_3} w \, \mathrm{d}y$$

we have:

$$w = \int (\sigma_y \, \mathrm{d}\varepsilon_y) \qquad \text{and} \qquad \sigma_y = E\epsilon_y = E\frac{\nu_0}{h}$$

So:

$$w = \int E\frac{\nu_0}{h} \, d(\frac{\nu_0}{h}) = \int \frac{E}{h^2}\nu_0 \, d(\nu_0) = \frac{1}{2}E(\frac{\nu_0}{h})^2$$

Thus:

$$J = \int_0^{2h} \frac{1}{2}E(\frac{\nu_0}{h})^2 \, dy = \frac{1}{2}E(\frac{\nu_0}{h})^2 2h = E\frac{\nu_0^2}{h}$$

According to the eq 3.44(in Book): $J_c = \sigma_y.\delta_c$

$$\frac{E\nu_{0,crit}^2}{h} = \sigma_y.\delta_c$$

$$\Longrightarrow \quad \nu_{0,\mathrm{crit}} = \sqrt{\frac{\sigma_y}{E}\delta_c h}$$