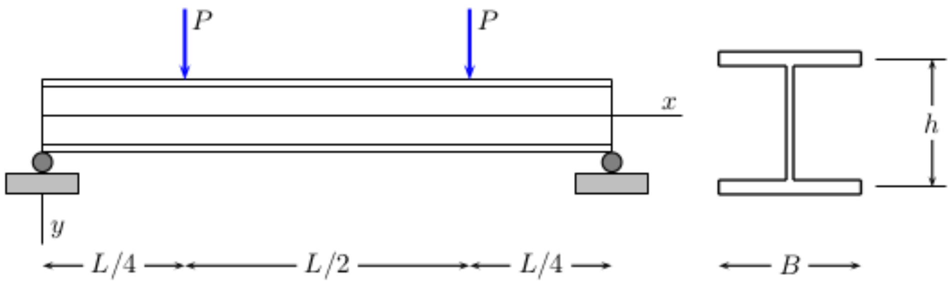
RAK-33060 Fracture mechanics and fatigue

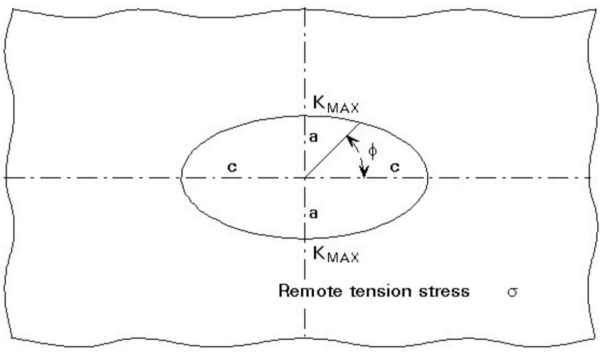
1. Home exercise

1. Consider a simply supported beam as shown below. Assume that there exist a semi-elliptical surface crack at the bottom of the lower flange (between the two external loads) with length $c = B/10$, $a = c/2$ (see Fig. 2.19 in Anderson's book). Calculate the maximum allowable load P if the fracture toughness is $K_{\mathrm{Ic}} = 60\,\mathrm{MPam}^{1/2}$. Dimensions of the profile are: $B = 220$ mm, $h = 580$ mm, flange thickness $t_{\mathrm{f}} = 20$ mm, web thickness $t_{\mathrm{w}} = 10$ mm. Length of the beam is $L = 10$ m and the Young's modulus is $E = 200$ GPa.

   In the analysis you can assume that bending is taken only by the flanges and the stress is constant in the flanges.



2. Determine the stress intensity factor K_{I} for an eliptical planar crack in an infinite body under uniaxial normal stress σ perpendicular to the crack plane. Use the Griffith energy approach assuming that stresses are relaxed in an ellipsoid around the crack.



   The analytical solution of this problem has given by Green and Sneddon 1950 and it is

$$K_{\mathrm{I}} = \frac{\sigma\sqrt{\pi a}}{E(m)}\left(1 + m^2\cos^2\phi\right)^{1/4},$$

   where $m = \sqrt{1 - a^2/c^2}$, a, c are the half-axis lengths of the ellipse and $E(m)$ is the complete elliptic integral of the second kind

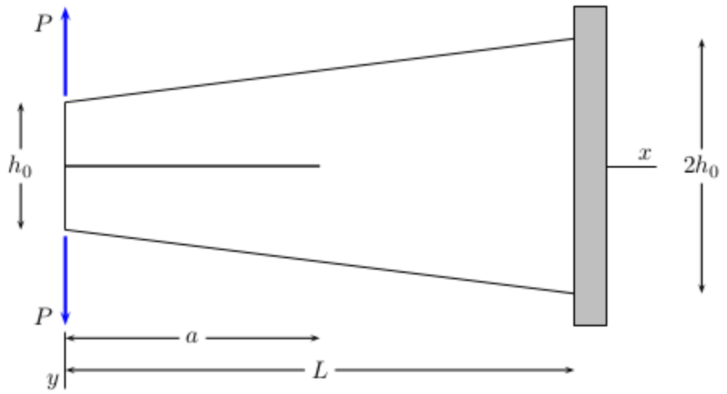
$$E(m) = \int_0^{\pi/2} \sqrt{1 - m^2\sin^2\theta}\,\mathrm{d}\theta.$$

   Draw the stress intensity factor along the crack front and compare to the simple solution you found. What is the error in your solution in comparison to the maximum stress intensity factor.

3. Calculate the stress intensity factor of the centrally cracked cantilever beam. The height of the beam varies as $h(x) = h_0(1 + x/L)$ and the width is a constant B. The ratio $L/a = 2$. Young's modulus of the beam is E.

   Hint. Remember the Simpsons rule for the numerical evaluation for definite integrals:

$$\int_a^b f(x)\mathrm{d}x = \frac{a-b}{6}\left(f(a) + 4f(\tfrac{1}{2}(1+b)) + f(b)\right).$$

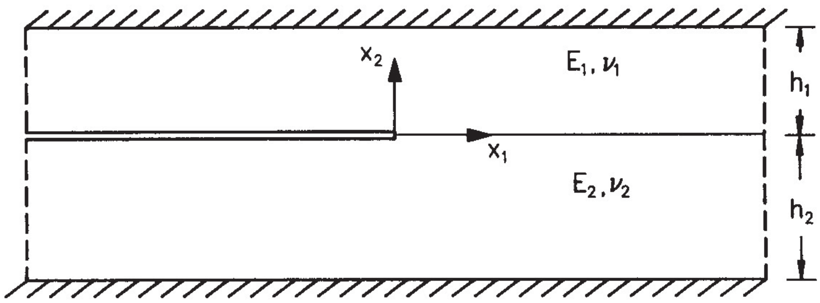


4. The crack growth resistance curve of a certain material at a thickness 2 mm is expressed by

$$R = \frac{K_{\mathrm{Ic}}^2}{E} + C_0\left(\frac{\Delta a}{B}\right)^{1/4}.$$

   Consider a center cracked steel plate of width $W = 10$ cm and thickness $B = 2$ mm with a crack of length $a = 1$ mm. Calculate the length of stable crack growth, the critical crack length and the critical stress at instability. Material data are $K_{\mathrm{Ic}} = 95$ MPa$\sqrt{\mathrm{m}}$, $C_0 = 1$ MJ/m^2 (note: J/m^2 = N/m).

5. Two infinite layers of heights h_1 and h_2 and large thicknesses are made of different materials with moduli of elasticity and Poisson's ratios E_1, ν_1 and E_2, ν_2, respectively. The layers are joined accross their interface forming a semi-infinite crack, and rigidly clamped along their bases at $x_2 = h_1$ and $x_2 = -h_2$. The upper and lower ends are moved in the positive and negative x_1-direction over distances u_0, respectively. Determine the value of J-integral.



To be returned at latest on Thursday 7.11.2019 via the Moodle system.