RAK-33060 Fracture mechanics and fatigue

4. Exercise, energy principles, weight functions

1. Use the principle of virtual work to solve the following beam problem

\[ EI \frac{d^4 v}{dx^4} = q, \quad x \in (0, L), \]

with boundary conditions \( v(0) = v'(0) = 0, v(L) = \Delta \) and \( M(L) = 0 \). For the prescribed displacement \( \Delta \) choose \( \Delta = \alpha q L^4 / (12 EI) \), where \( \alpha \) is a dimensionless parameter.

(a) Construct the simplest possible trial function for \( v(x) \).
(b) Construct the corresponding virtual displacement \( \delta v(x) \).
(c) Solve the problem and plot the deflection \( v \) and the bending moment \( M(x) \) for \( \alpha = 0.02 \).

2. Determine the weight function \( h(x) \) and give the expression for the stress-intensity factor of the right crack tip in the split beam loaded with arbitrarily distributed load \( q(x) \) (force/length) according to the figure (b). Width of the beam is \( b \). Utilise the solution to the problem in figure a and assume plane stress conditions. Calculate \( K_I \) for the case \( q(x) = q_0 x / a \).

3. A beam of width \( b \) has a crack according to the figure below and is loaded in pure bending. Determine the stress-intensity factor when \( h \gg 2a \). The material is linearly elastic.
The weight function is
\[ h(x) = \frac{1}{\sqrt{\pi a}} \sqrt{\frac{a + x}{a - x}}. \]

4. A short cantilever beam of a linearly elastic material contains a crack oriented according to the figure below. Compute an approximate expression for the stress intensity factor if the height of the web of the I-beam can be regarded as much larger than the crack size.