RAK-33060 Fracture mechanics and fatigue

4. Exercise, energy principles, weight functions

1. Use the principle of virtual work to solve the following beam problem

$$EI\frac{\mathrm{d}^4 v}{\mathrm{d}x^4} = q, \quad x \in (0, L),$$

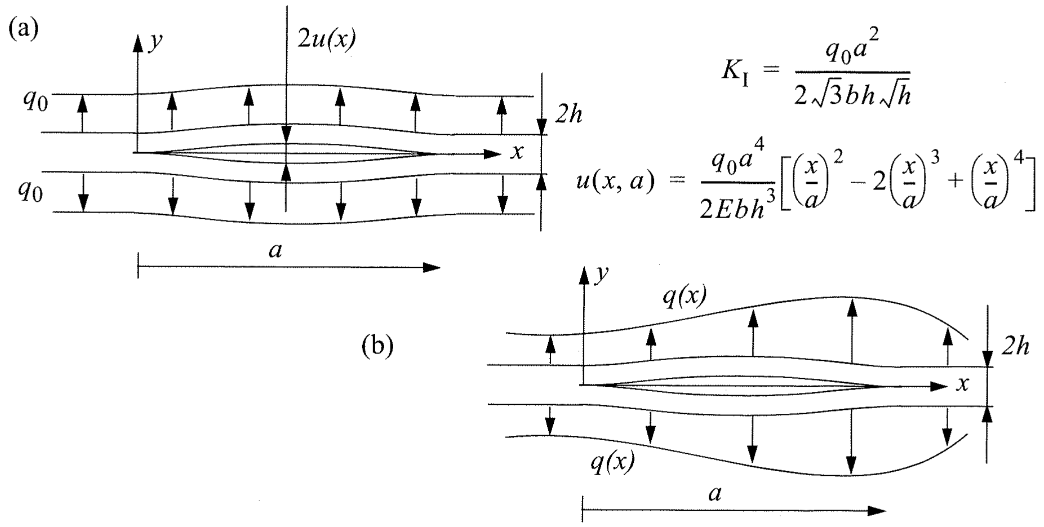
   with boundary conditions $v(0) = v'(0) = 0$, $v(L) = \Delta$ and $M(L) = 0$. For the prescribed displacement Δ choose $\Delta = \alpha q L^4/(12EI)$, where α is a dimensionless parameter.

   (a) Construct the simplest possible trial function for $v(x)$.

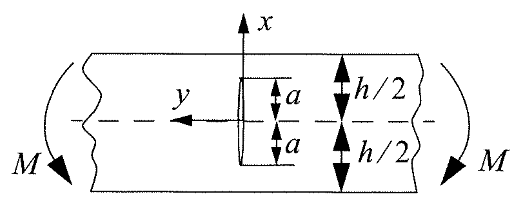
   (b) Construct the corresponding virtual displacement $\delta v(x)$.

   (c) Solve the problem and plot the deflection v and the bending moment $M(x)$ for $\alpha = 0.02$.

2. Determine the weight function $h(x)$ and give the expression for the stress-intensity factor of the right crack tip in the split beam loaded with arbitrarily distributed load $q(x)$ (force/length) according to the figure (b). Width of the beam is b. Utilise the solution to the problem in figure a(and assume plane stress conditions. Calculate K_I for the case $q(x) = q_0 x/a$.



3. A beam of width b has a crack according to the figure below and is loaded in pure bending. Determine the stress-intensity factor when $h \gg 2a$. The material is linearly elastic.



The weight function is

$$h(x) = \frac{1}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}.$$

4. A short cantilever beam of a linearly elastic material contains a crack oriented according to the figure below. Compute an approximate expression for the stress intensity factor if the height of the web of the I-beam can be regarded as much larger than the crack size.