3. Exercise

1. A large plate has a crack inclined by an angle \( \alpha \) w.r.t. the horizontal line. The length of the crack is \( 2a \). The plate is loaded by a horizontal tensile stress \( \sigma_x = \sigma_\infty \). Determine the stress intensity factors at the crack tip. At the end of this paper there is a table of stress intensity factors for basic loading cases.

![Diagram of a large plate with a crack inclined by an angle \( \alpha \)](image)

2. Investigate the previous structure. Assume that the fracture occurs if

\[
\left( \frac{K_I}{K_{Ic}} \right)^2 + \left( \frac{K_{II}}{K_{IIc}} \right)^2 = 1,
\]

where \( K_{Ic} \neq K_{IIc} \). Investigate which angles \( \alpha \) are the most dangerous as a function of the ratio \( K_{Ic}/K_{IIc} \).

3. A crack grows along the interface in a bi-material bar of width \( B \) under a tensile force \( F \) (figure below on the LHS).

(a) Determine the crack driving force \( G \) using simple bar model.
(b) Determine \( K_{II} \) for the case \( E_1 = E_2 \) under the assumption that pure mode II and plane stress is present.

![Diagram of a bi-material bar with a crack](image)

4. Calculate the crack driving force \( G \) and the stress intensity factor \( K_I \) for the structure shown above on the right hand side. Assume the state of plane strain and that \( h \ll a \).
5. Calculate the crack deflection angle $\varphi$ for the two configurations shown below. Use the criterion of maximum circumferential stress and assume $\tau_0 = \sigma_0/2$.

![Diagrams of crack configurations](image)

<table>
<thead>
<tr>
<th></th>
<th>Diagram</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image" alt="Diagram" /></td>
<td>[ \begin{bmatrix} K_I \ K_{II} \end{bmatrix} = \begin{bmatrix} \sigma \ \tau \end{bmatrix} \frac{1}{\sqrt{\pi a}} ]</td>
</tr>
<tr>
<td>2</td>
<td><img src="image" alt="Diagram" /></td>
<td>[ \begin{bmatrix} K_I^+ \ K_{II}^+ \end{bmatrix} = \begin{bmatrix} P \ Q \end{bmatrix} \frac{1}{\sqrt{\pi a}} \sqrt{\frac{a \pm b}{a \pm b}} ]</td>
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<tr>
<td>3</td>
<td><img src="image" alt="Diagram" /></td>
<td>[ \begin{bmatrix} K_I \ K_{II} \end{bmatrix} = \begin{bmatrix} \sigma \ \tau \end{bmatrix} \sqrt{2b \tan \frac{\pi a}{2b}} ]</td>
</tr>
<tr>
<td>4</td>
<td><img src="image" alt="Diagram" /></td>
<td>[ \begin{bmatrix} K_I \ K_{II} \end{bmatrix} = \begin{bmatrix} P \ Q \end{bmatrix} \frac{2}{\sqrt{2\pi b}} ]</td>
</tr>
<tr>
<td>5</td>
<td><img src="image" alt="Diagram" /></td>
<td>[ K_I = 1.1215 \sigma \sqrt{\pi a} ]</td>
</tr>
</tbody>
</table>
| 6 | ![Diagram](image) | \[ K_I = \sigma \sqrt{\pi a} F_I(a/b) \]  
\[ F_I = \frac{1 - 0.025(a/b)^2 + 0.06(a/b)^4}{\sqrt{\cos (\pi a/2b)}} \] |