Continuum mechanics - 2020

2. home exercise – kinematics, mass conservation

1. A motion is given in the material description (Lagrangian formulation) as
   \[x_1 = X_1 \exp(t/t_0) + X_3[\exp(t/t_0) - 1],\]
   \[x_2 = X_2 + X_3[\exp(t/t_0) - \exp(-t/t_0)],\]
   \[x_3 = X_3.\]
   (a) Find out the motion in the spatial description (Eulerian formulation).
   (b) Determine the velocity- and acceleration fields both in the material and spatial description.

2. A material body \(B\) is a cube in the initial undeformed configuration \(0 \leq X_1, X_2, X_3 \leq L\). Assuming that the body deforms uniformly (homogeneous deformation) occupying the volume \(0 \leq x_1 \leq L + a, \ 0 \leq x_2 \leq L + b\) ja \(0 \leq x_3 \leq L\). Determine the deformation mapping, displacement vector, deformation gradient \(F\), the right Cauchy-Green deformation tensor \(C\), right stretch tensor \(U\), the Green-Lagrange strain tensor \(E\) and the infinitesimal strain tensor \(\varepsilon\). Calculate by using
   (a) the Green-Lagrange strain tensor \(E\),
   (b) the infinitesimal strain tensor \(\varepsilon\), and
   (c) the Biot strain tensor \(E^{(1)} = U - I\)
   the change in length between the points \(A\) and \(B\) and also between \(A\) and \(C\). The coordinates of the points \(A, B\) and \(C\) in the undeformed configuration are: \(A = (0,0,0), B = (L,L,0)\) and \(C = (L,0,0)\). Check the result by using elementary geometry. What are your comments on these results.

3. A spatial velocity field of a plane motion has components of the form
   \[v_1 = (\alpha x_1 - \beta x_2)t, \quad v_2 = \beta x_1 - \alpha x_2, \quad v_3 = 0,\]
   where \(\alpha\) and \(\beta\) are positive constants. Assume that the spatial mass density \(\rho\) is independent of the current position \(x\) so that \(\text{grad} \rho = 0\).
   (a) Express \(\rho\) so that the continuity mass equation is satisfied.
   (b) Find a condition for which the given motion is isochoric.

4. Two motions of a continuum body are given in the form
   \[x = (1 + \alpha(t)t)X, \quad x = X + \alpha(t)t^2 (e_1 \otimes e_2 - e_2 \otimes e_1) X,\]
   with the scalar function \(\alpha(t)\) and the set \(\{e_a\}, a = 1, 2, 3,\) of orthogonal unit vectors.
   Find expression for the spatial mass density \(\rho\) in terms of \(\rho_0\) so that the continuity mass equation is satisfied.

Last day to return is Tuesday 29.9.2020.