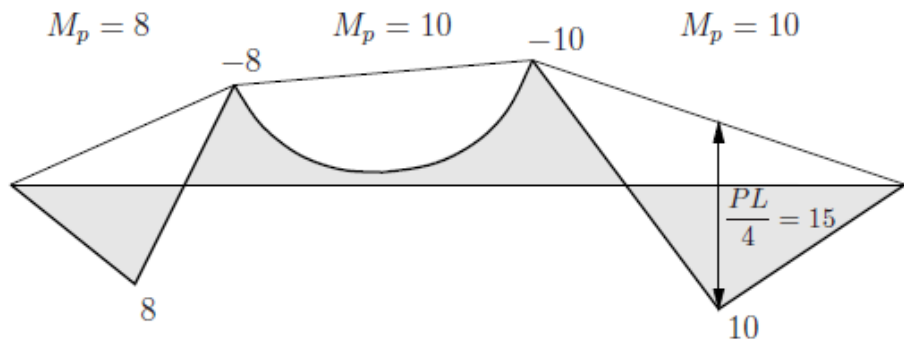
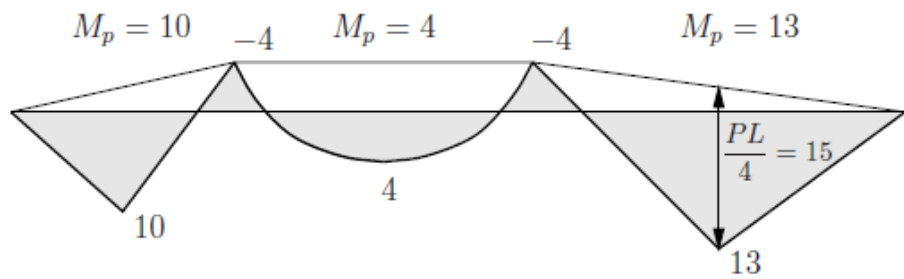
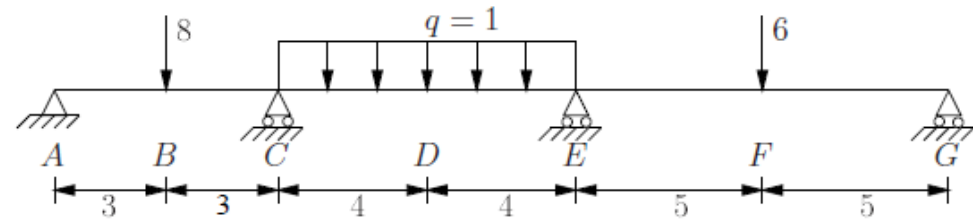


33001 Rakenteiden plastisuusmallit

Minimipainomitoitus

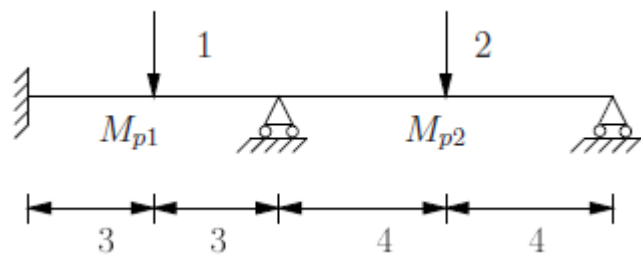


Yksikköpaino: $g = kM_p^n$

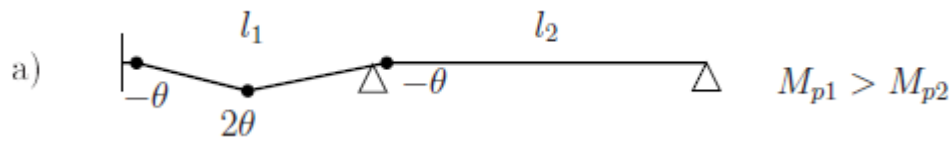
Jos linearisoidaan: $g = kM_p$

Rakenteen paino: $G = k \sum_i M_{pi} L_i$

Voidaan käyttää lineaarista painofunktioita yksinkertaisille palkki- ja kehärakenteille



$$G = M_{p1}L_1 + M_{p2}L_2 = 6M_{p1} + 8M_{p2}.$$

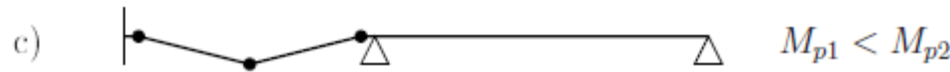


$$a : \theta M_{p1} + 2\theta M_{p1} + \theta M_{p2} = 3 \cdot \theta \cdot 1$$

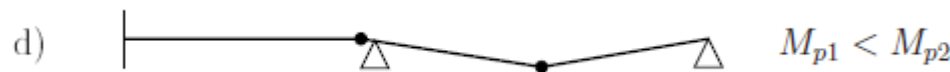
$$a : 3M_{p1} + M_{p2} = 3$$



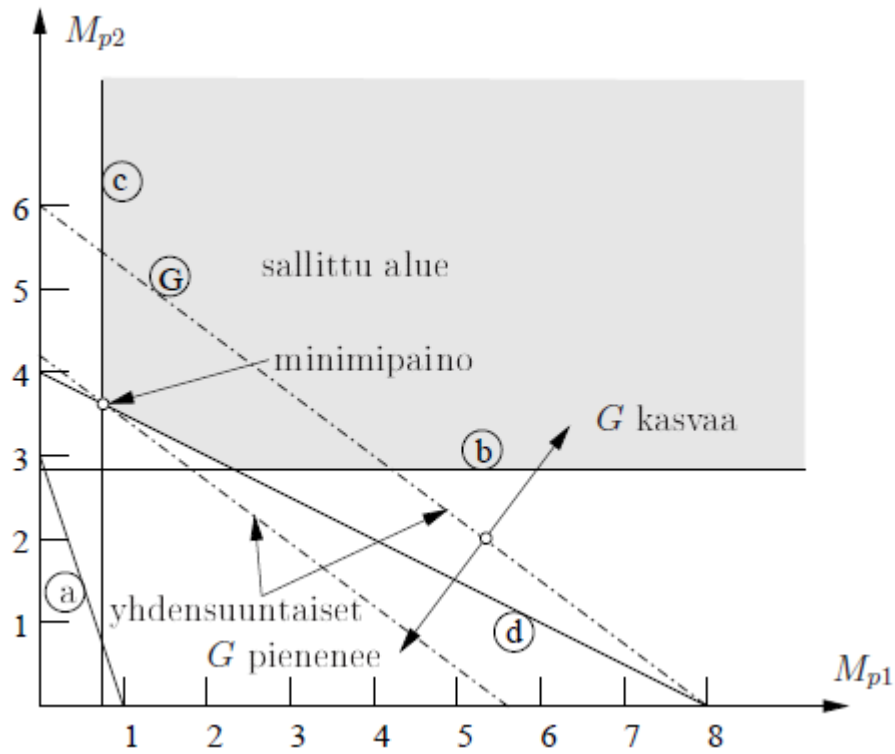
$$b : 3M_{p2} = 8,$$



$$c : 4M_{p1} = 3$$

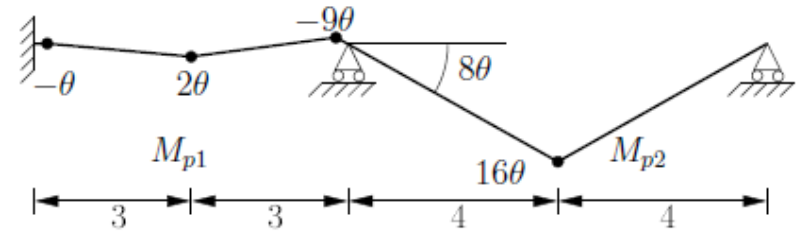


$$d : M_{p1} + 2M_{p2} = 8$$

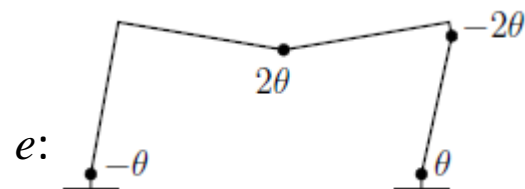
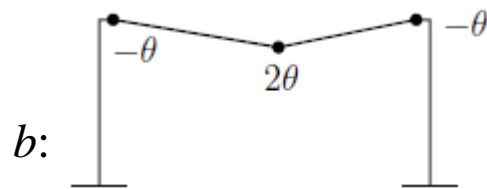
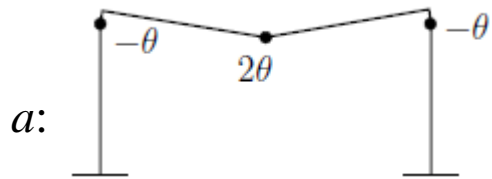
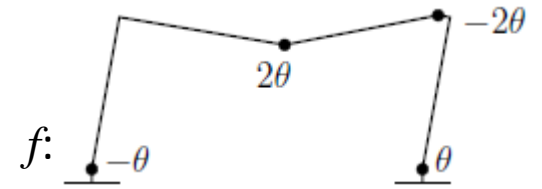
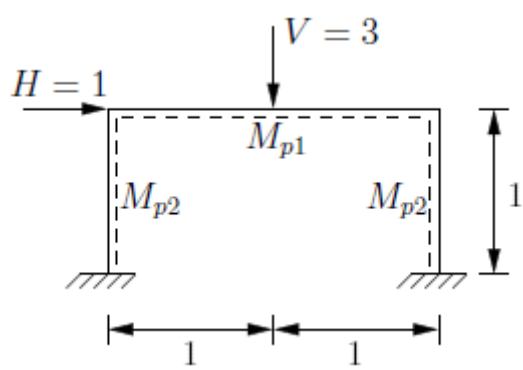


$$c : 4M_{p1} = 3$$

$$d : M_{p1} + 2M_{p2} = 8$$



Palkin minimipainoa vastaava mekanismi eli painosuoramekanismi



$$G = k(2M_{p1} + 2M_{p2})$$

$$a : 2M_{p1} + 2M_{p2} = 3$$

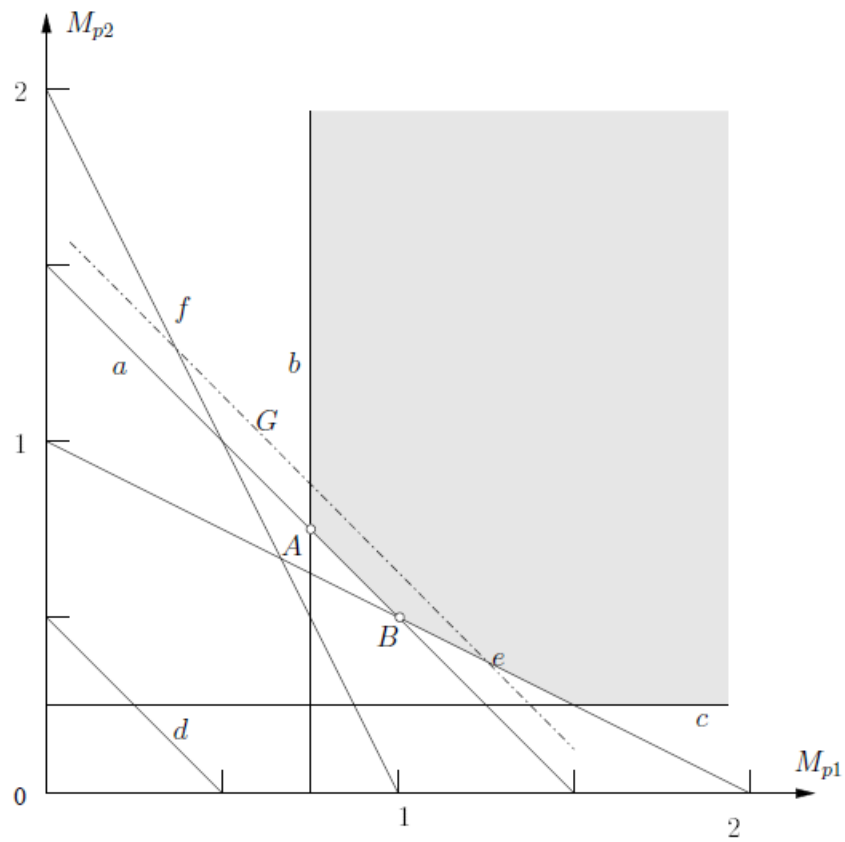
$$b : 4M_{p1} = 3$$

$$c : 4M_{p2} = 1$$

$$d : 2M_{p1} + 2M_{p2} = 1$$

$$e : 2M_{p1} + 4M_{p2} = 4$$

$$f : 4M_{p1} + 2M_{p2} = 4$$



Jana AB on minimipainoratkaisu

$$\frac{(\sum \theta)_1}{(\sum \theta)_2} = \frac{2\theta}{2\theta} = \frac{L_1}{L_2} = \frac{2}{2}$$

Mekanismiehto:

$$\sum_j M_{pj} \phi_j = \sum_i F_i \Delta_i$$

$$\phi_j = \alpha L_j$$

$$\phi_j = \sum_k \theta_{jk}$$

Jos taivutusmomentit toteuttavat tasapainoehdot ja myötöehdot, niin rakenteen paino on mahdollisimman pieni (minimipaino) annetulle kuormitukselle.

Todistus:

$$G^* = k \sum_j M_{pj}^* L_j$$

$$\frac{\alpha}{k} G = \sum_i F_i \Delta_i$$

$$\sum_j (\sum_k M_{jk} \theta_{jk}) = \sum_i F_i \Delta_i$$

$$G^* \geq G$$

$$-M_{pj}^* \leq M_{jk} \leq M_{pj}^*$$

$$\sum_j M_{pj}^* \phi_j \geq \sum_i F_i \Delta_i$$

$$\phi_j = \alpha L_j \text{ ja } G^* = k \sum_j M_{pj}^* L_j$$

$$\frac{\alpha}{k} G^* \geq \sum_i F_i \Delta_i$$

Mitoitus antaa minimipainon, jos se toteuttaa

- tasapainoehdot
- myötöehdot
- mekanismiehdon
- ehdon (9.27) $\phi_j = \alpha L_j$

Minimipainon alaraja

Lause 9.2 *Mekanismiehdon ja plastisten nivelkiertymien ehdon toteuttava mitoitus antaa rakenteen minimipainolle alarajan.*

$$\phi_j^* = \alpha L_j$$

$$\sum_j M_{pj}^* \phi_j^* = \sum_i F_i \Delta_i^*$$

$$G^* = k \sum_j M_{pj}^* L_j = \frac{k}{\alpha} \sum_i F_i \Delta_i^*$$

$$-M_{pj} \leq M_{jk} \leq M_{pj}$$

$$\sum_j \left(\sum_k M_{jk} \theta_{jk}^* \right) = \sum_i F_i \Delta_i^*$$

$$\sum_j \left(\sum_k M_{jk} \theta_{jk}^* \right) \leq \sum_i M_{pj} \phi_j^*$$

$$\Rightarrow \sum_j M_{pj} \phi_j^* \geq \sum_i F_i \Delta_i^*$$

$$G = k \sum_j M_{pj} L_j = \frac{k}{\alpha} \sum_j M_{pj} \phi_j^* \geq \frac{k}{\alpha} \sum_i F_i \Delta_i^*$$

$$G^* \leq G$$

