33001 Rakenteiden plastisuusmallit

Taipumien määritys rajatilassa

$$\varphi_{ij} = \alpha_{ij} M_{ij} - \beta_{ij} M_{ji} + \psi_{ij} + \alpha_{ij}^0$$

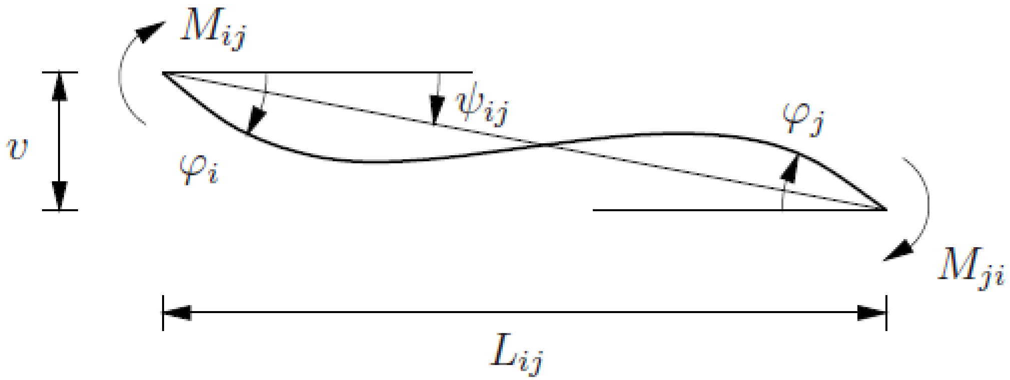
$$\varphi_{ji} = -\beta_{ji} M_{ij} + \alpha_{ji} M_{ji} + \psi_{ij} + \alpha_{ji}^0$$

$$\psi_{ij} = \frac{v}{L_{ij}}$$

$$\alpha_{ij} = \alpha_{ji} = \frac{L_{ij}}{3EI}$$

$$\beta_{ij} = \beta_{ji} = \frac{L_{ij}}{6EI}$$

$$\alpha_{ij}^0 = -\alpha_{ji}^0 = \frac{qL_{ij}^3}{24EI}$$

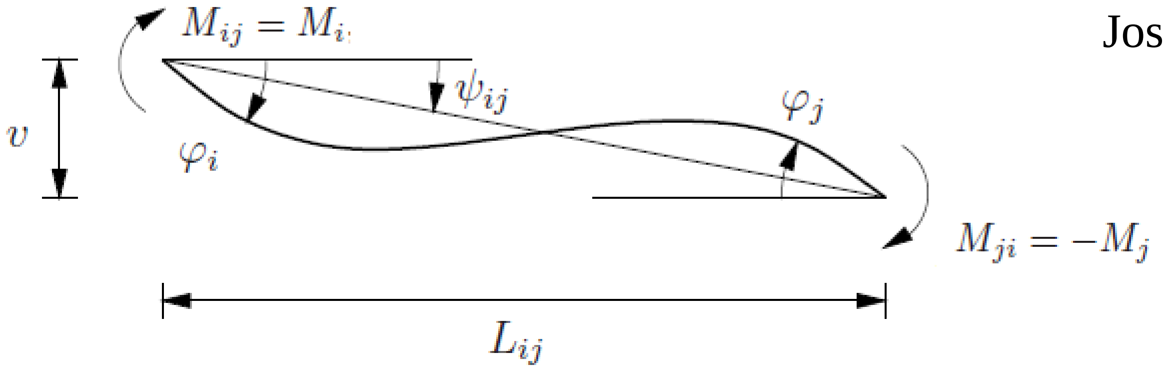


Jos **ei** niveltä: $\varphi_{ji} = \varphi_{jk}$

Jos nivel: $\theta_j = \varphi_{ji} - \varphi_{jk}$

$$\varphi_i = \frac{L_{ij}}{3EI} M_i + \frac{L_{ij}}{6EI} M_j + \frac{v}{L_{ij}} + \frac{qL_{ij}^3}{24EI}$$

$$\varphi_j = -\frac{L_{ij}}{6EI} M_i - \frac{L_{ij}}{3EI} M_j + \frac{v}{L_{ij}} - \frac{qL_{ij}^3}{24EI}$$



Jos **ei** niveltä: $\varphi_{ji} = \varphi_{jk}$

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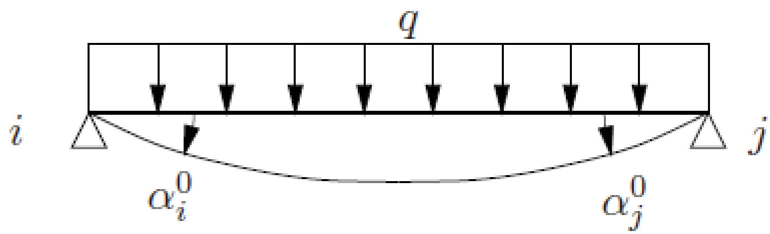
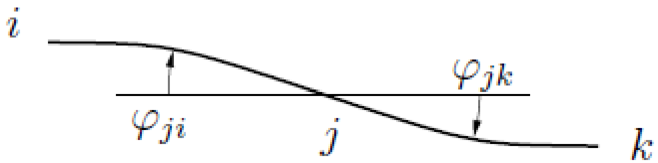
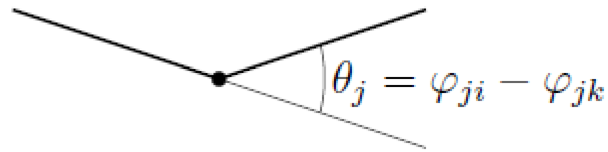
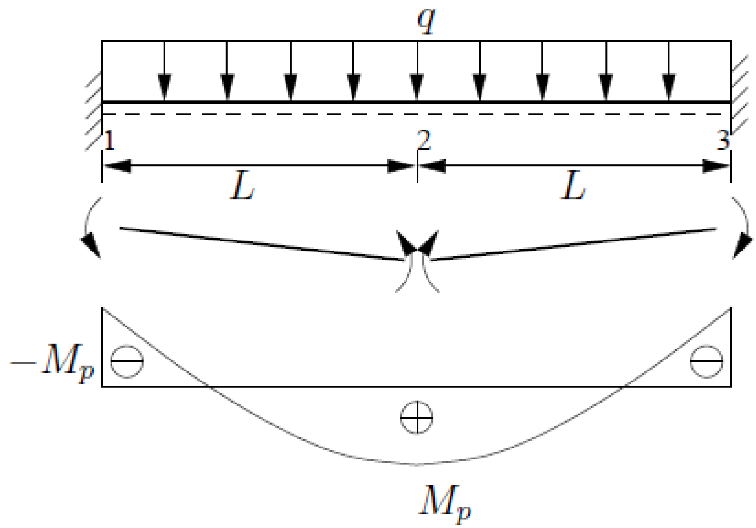
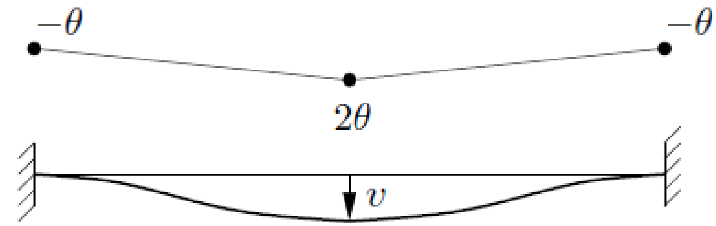




$$\varphi_{ji} = \varphi_{jk}$$







$$M_{12} = -M_p, \quad M_{21} = -M_p,$$

$$M_{23} = M_p, \quad M_{32} = M_p.$$

$$M_{ij} = M_i, \quad M_{ji} = -M_j$$

$$4M_p\theta = \frac{1}{2}q2L \cdot L\theta,$$

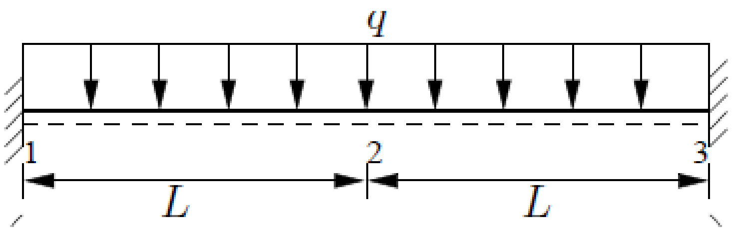
$$q_p = 4\frac{M_p}{L^2}$$

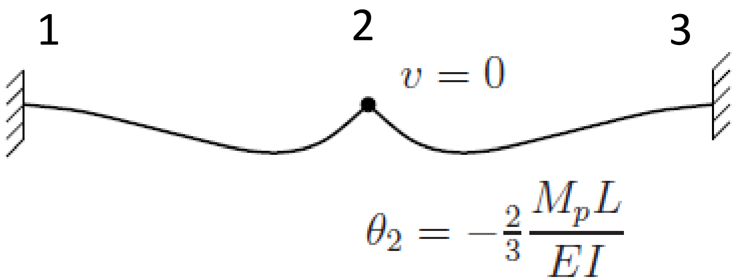
$$\varphi_{12} = \frac{v}{L}$$

$$\varphi_{21} = -\frac{L}{3EI}M_p + \frac{v}{L}$$

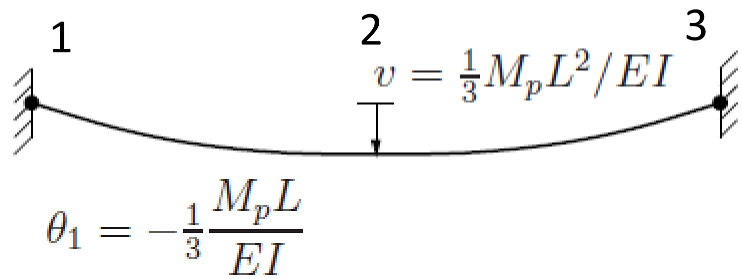
$$\varphi_{23} = \frac{L}{3EI}M_p - \frac{v}{L}$$

$$\varphi_{32} = \frac{v}{L}$$



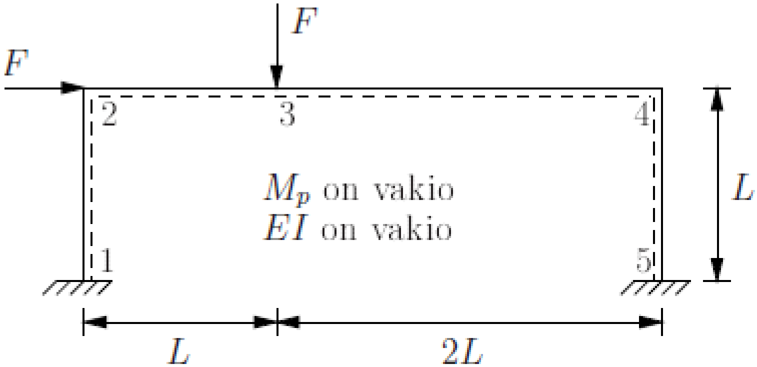


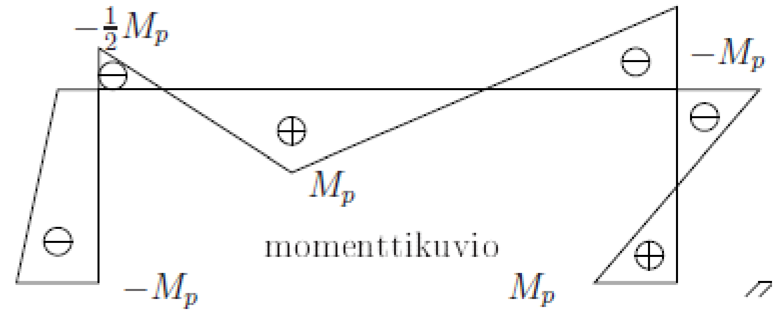
a) Viimeinen nivel syntyy pisteeseen 1

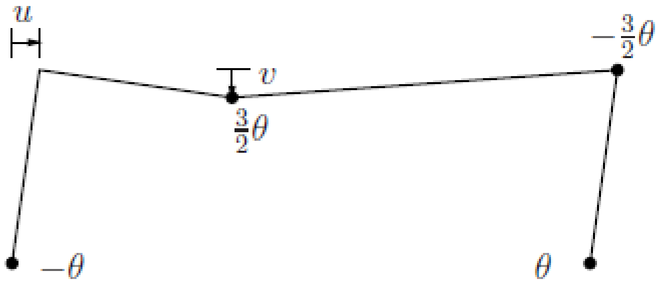


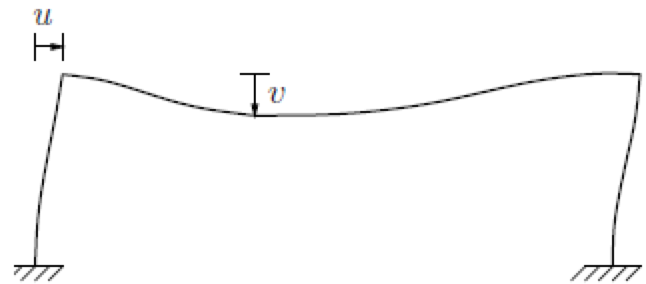
b) Viimeinen nivel syntyy pisteeseen 2

Lause 7.1 *Jos kuormituksen aikana mikään syntynyt plastinen nivel ei ole palautunut (alkanut kiertyä vastakkaiseen suuntaan), niin suurin siirtymä on oikea, kun siirtymät on määritetty olettaen vuoronperään kunkin plastisen nivelen syntyneen viimeiseksi.*









$$5M_p\theta = FL\theta + FL\theta \qquad \Rightarrow \qquad F_p = \frac{5}{2}\frac{M_p}{L}$$

a) Nivel 1 viimeinen Ok

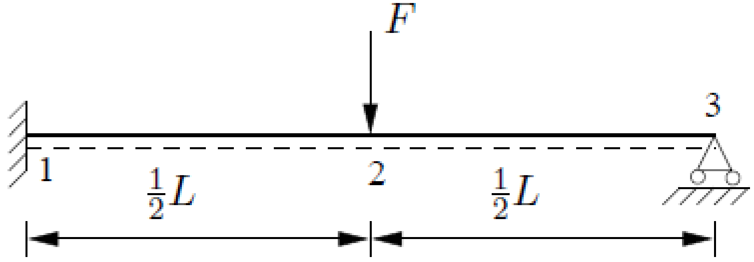
b) Nivel 3 viimeinen $\theta_1 = -\varphi_{12} = \dfrac{13}{36}\dfrac{M_pL}{EI} > 0$

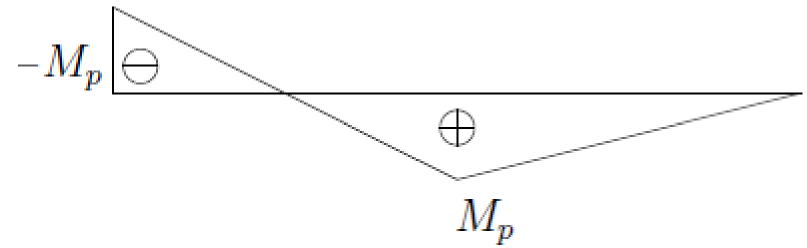
c) Nivel 4 viimeinen $\theta_1 = -\varphi_{12} = \dfrac{7}{36}\dfrac{M_pL}{EI} > 0.$

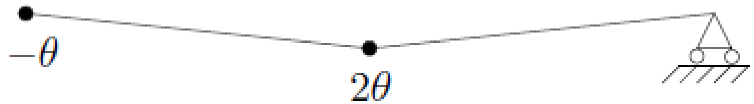
d) Nivel 5 viimeinen $\theta_1 = -\varphi_{12} = \dfrac{3}{12}\dfrac{M_pL}{EI} > 0$

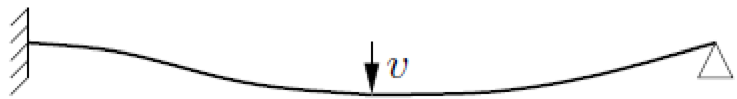
	a	b	c	d	max
$u/(M_pL^2/36EI)$	15	2	8	6	15
$v/(M_pL^2/36EI)$	27	14	20	18	27

$$u = \frac{5}{12}\frac{M_pL^2}{EI} \quad \text{ja} \quad v = \frac{9}{12}\frac{M_pL^2}{EI}$$









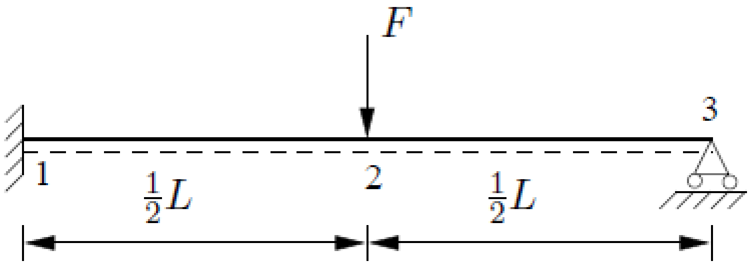
$$\varphi_{ij} = \beta(2M_i + M_j) + \psi_{ij} + \alpha^0_{ij},$$

$$\varphi_{ji} = -\beta(M_i + 2M_j) + \psi_{ij} + \alpha^0_{ji}$$

$$\beta = \frac{L}{6EI}$$

$$3M_p\theta = F\tfrac{1}{2}L\theta$$

$$F_p = 6\frac{M_p}{L}$$



$$\alpha_{ij}^{0} = 0$$

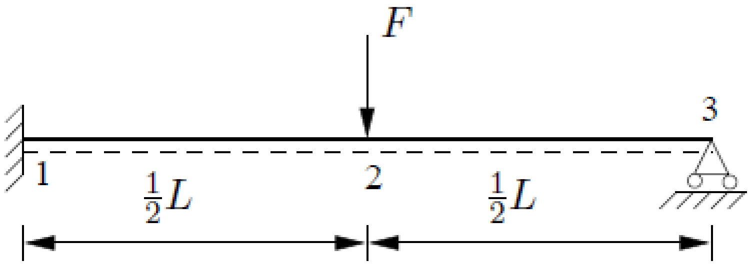
$$\beta = \frac{L}{12EI}$$

$$\psi_{12} = \psi_{21} = \frac{2v}{L} \qquad \psi_{23} = -\frac{2v}{L}$$

$$\varphi_{12} = \beta(-2M_p + M_p) + \frac{2v}{L} = -\beta M_p + \frac{2v}{L}$$

$$\varphi_{21} = -\beta(-M_p + 2M_p) + \frac{2v}{L} = -\beta M_p + \frac{2v}{L}$$

$$\varphi_{23} = \beta(2M_p) - \frac{2v}{L} = 2\beta M_p - \frac{2v}{L}$$



a) Viimeinen nivel syntyy pisteeseen 1

$$\theta_1 = -\varphi_{12} = 0$$

$$-\beta M_p + \frac{2v}{L} = 0 \quad \Rightarrow \quad v = \tfrac{1}{2}\beta M_p L$$

$$\theta_2 = \varphi_{21} - \varphi_{23} = -\beta M_p + \frac{2v}{L} - 2\beta M_p + \frac{2v}{L}$$

$$= -\beta M_p + \beta M_p - 2\beta M_p + \beta M_p = -\beta M_p < 0$$

b) Viimeinen nivel syntyy pisteeseen 2

$$\theta_2 = \varphi_{21} - \varphi_{23} = 0$$

$$-\beta M_p + \frac{2v}{L} - 2\beta M_p + \frac{2v}{L} = 0$$

$$v = \frac{3}{4}\beta M_p L = \frac{3}{48}\frac{M_p L^2}{EI}$$

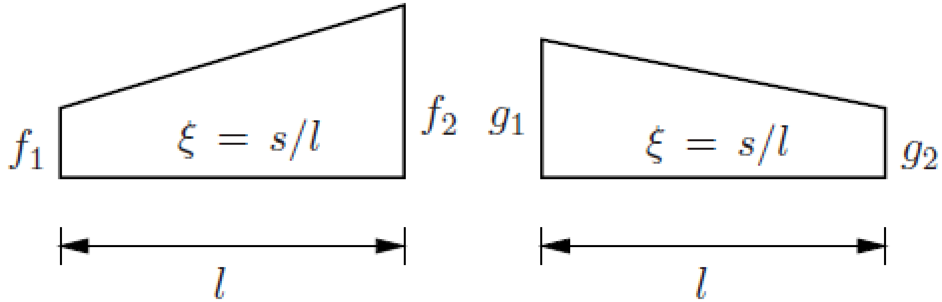
$$\theta_1 = -\varphi_{12} = \beta M_p - \frac{2v}{L} = -\frac{1}{2}\beta M_p < 0$$

Virtuaalisen voiman periaate

$$\tilde{1}\delta = \int \tilde{M}\frac{M}{EI}ds + \sum \tilde{M}_i\theta_i$$

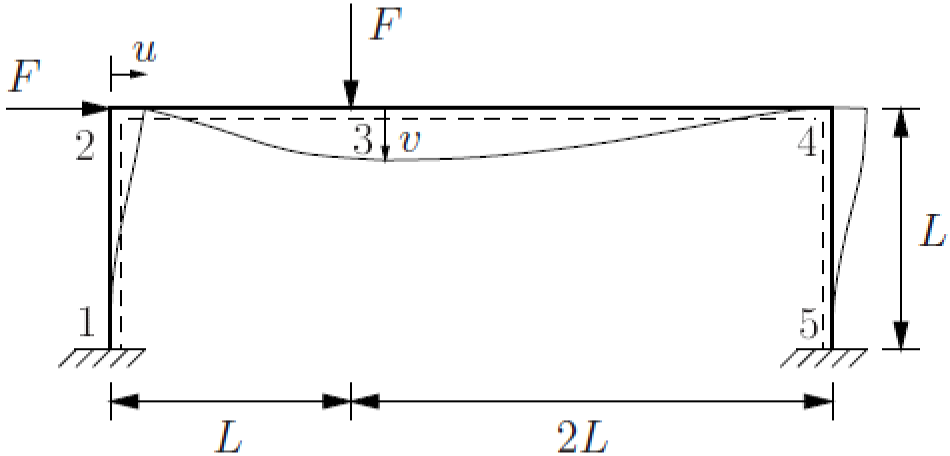
Jos plastinen nivel,
niin $\tilde{M}_i = 0$

$$\tilde{M}_k\theta_k = \delta - \int \tilde{M}\frac{M}{EI}ds$$

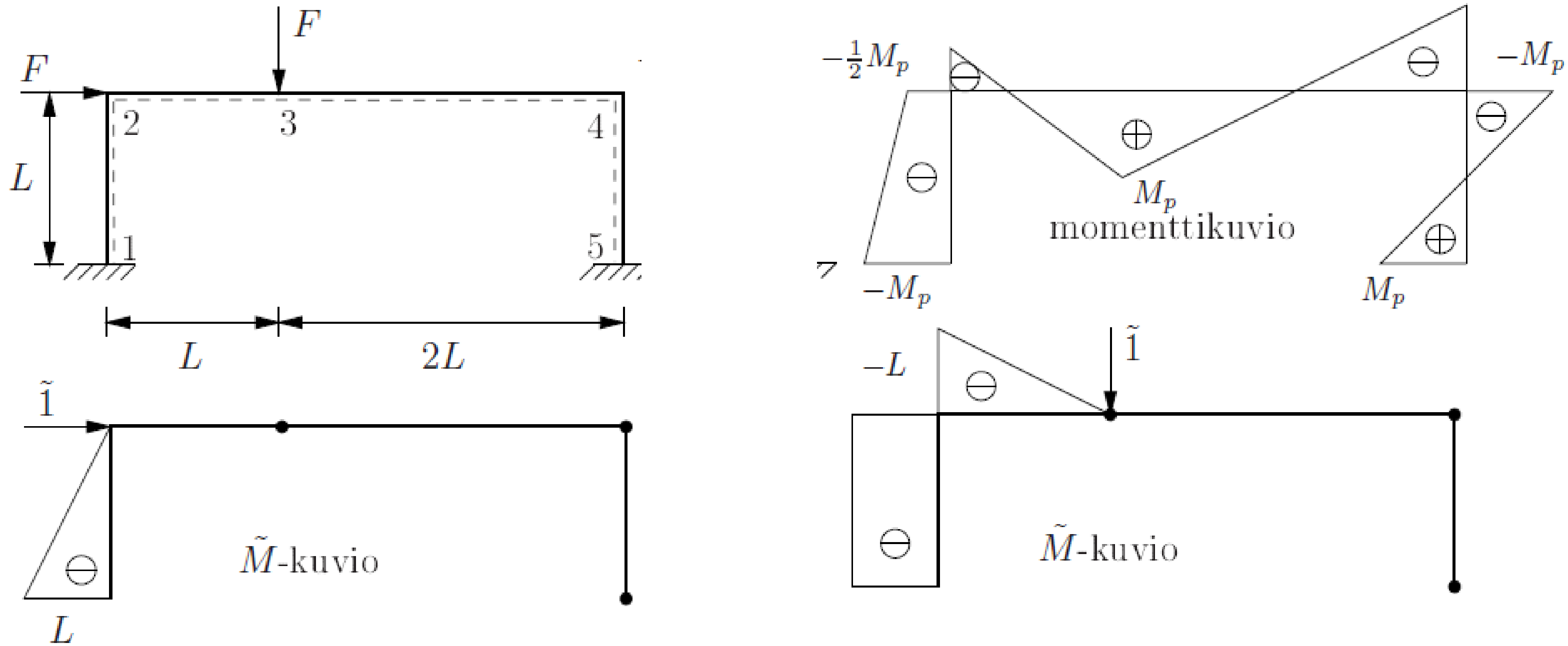


$$f(\xi) = (1-\xi)f_1 + \xi f_2; \qquad g(\xi) = (1-\xi)g_1 + \xi g_2;$$

$$\int_0^l f(s)g(s)ds = \frac{l}{6}[f_1(2g_1+g_2) + f_2(g_1+2g_2)]$$

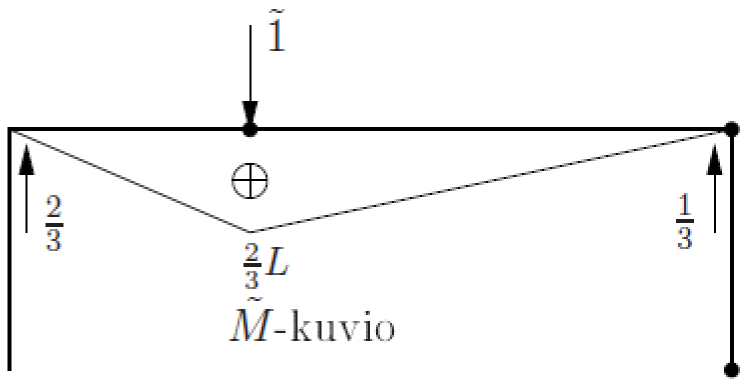


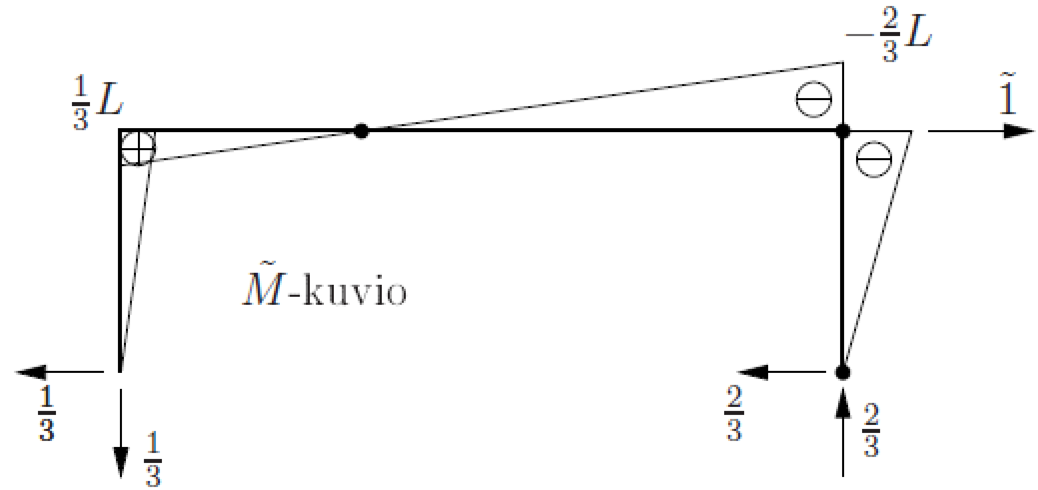
$$EIu = \int \tilde{M}\, M \, ds = \frac{L}{6}(-L)(-2M_p - \tfrac{1}{2}M_p) = \frac{5}{12} M_p L^2.$$

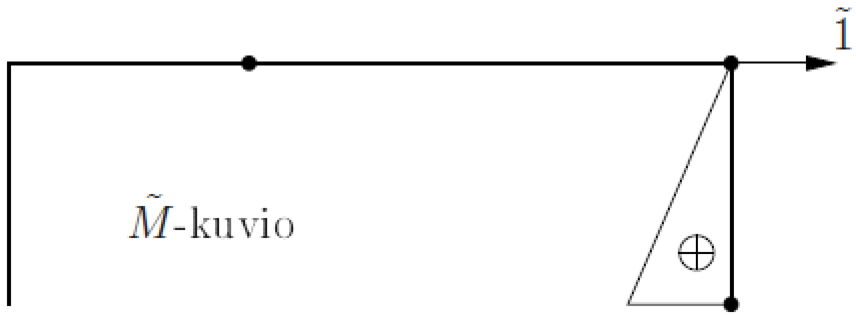


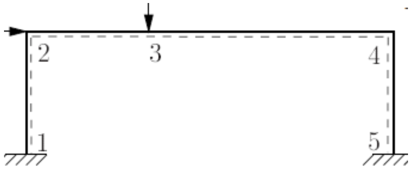
$$EIu = \int \tilde{M} M \, ds = \frac{L}{6}(-L)(-2M_p - \tfrac{1}{2}M_p) = \frac{5}{12}M_pL^2.$$

$$EIv = \int \tilde{M} M ds = \frac{9}{12}M_pL^2$$

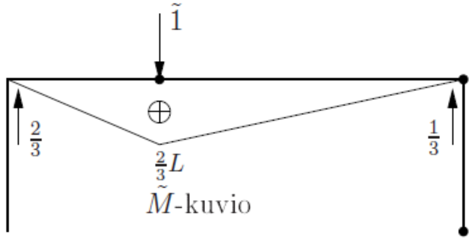

$\tilde{1}$
$\frac{2}{3}$
$\frac{1}{3}$
$\frac{2}{3}L$
$\tilde{M}$-kuvio

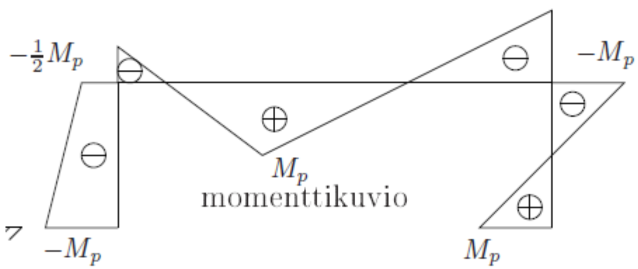

$-\frac{2}{3}L$
$\frac{1}{3}L$
$\tilde{1}$
$\tilde{M}$-kuvio
$\frac{1}{3}$
$\frac{1}{3}$
$\frac{2}{3}$
$\frac{2}{3}$


$\tilde{1}$
$\tilde{M}$-kuvio



$$\tilde{M}_3 \neq 0,\ \tilde{M}_4 = \tilde{M}_5 = 0$$





$$\tilde{M}_k \theta_k = \delta - \int \tilde{M} \frac{M}{EI} ds$$

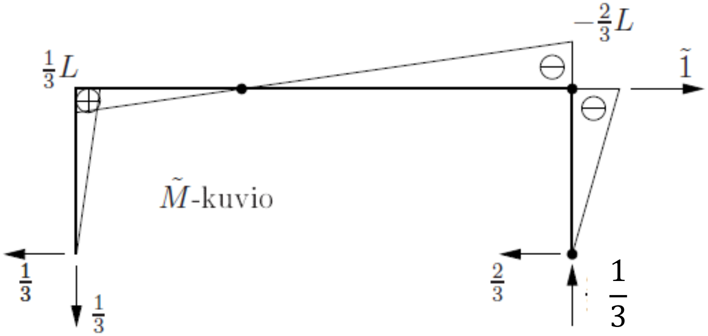
$$\frac{2}{3} L \theta_3 = \frac{9}{12} \frac{M_p L^2}{EI} - \int \tilde{M} \frac{M}{EI}\, ds$$

$$\int \tilde{M} M\, ds = \frac{L}{6} (\tfrac{2}{3}L)(-\tfrac{1}{2}M_p + 2M_p)$$

$$+ \frac{2L}{6} (\tfrac{2}{3}L)(2M_p - M_p)$$

$$= \left(\frac{3}{18} + \frac{4}{18} \right) M_p L^2 = \frac{7}{18} M_p L^2$$

$$\theta_3 = \left(\frac{27}{24} - \frac{21}{36} \right) M_p \frac{L}{EI} = \frac{13}{24} M_p \frac{L}{EI} > 0$$

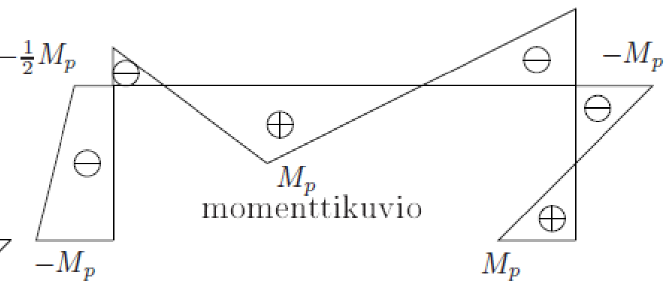


$$\tilde{1}\delta = \int \tilde{M}\frac{M}{EI}ds + \sum \tilde{M}_i\theta_i$$

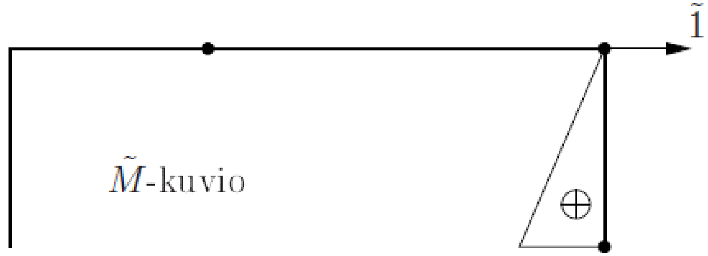
$$\tilde{1}u_4 = \int \tilde{M}\frac{M}{EI}ds + \tilde{M}_4\theta_4$$

$$\frac{2}{3}L\theta_4 = -\frac{5}{12}\frac{M_pL^2}{EI} + \frac{L}{6}\left(\frac{L}{3}\right)(-M_p - M_p)\frac{1}{EI} + \frac{L}{6}\left(\frac{L}{3}\right)(-M_p + M_p)\frac{1}{EI}$$

$$+ \frac{2L}{6}\left(-\frac{2L}{3}\right)(M_p - 2M_p)\frac{1}{EI} + \frac{L}{6}\left(-\frac{2}{3}L\right)(-2M_p + M_p)\frac{1}{EI},$$



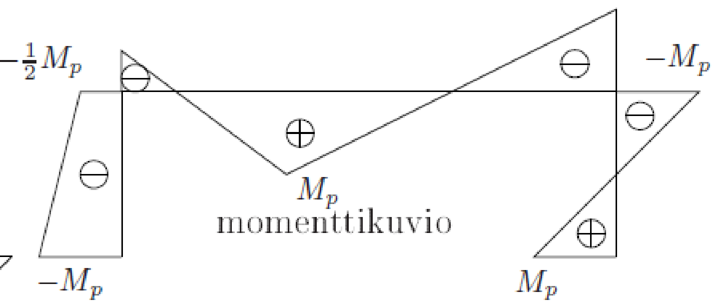
$$\theta_4 = -\frac{7}{24}\frac{M_pL}{EI} < 0$$



$$\tilde{1}\delta = \int \tilde{M}\frac{M}{EI}ds + \sum \tilde{M}_i\theta_i$$

$$\tilde{1}u_4 = \int \tilde{M}\frac{M}{EI}ds + \tilde{M}_5\theta_5$$

$$L\theta_5 = \frac{5}{12}\frac{M_pL^2}{EI} - \frac{L}{6}(L)(-M_p + 2M_p)\frac{1}{EI}$$



$$\theta_5 = \frac{3}{12}\frac{M_pL}{EI} > 0$$