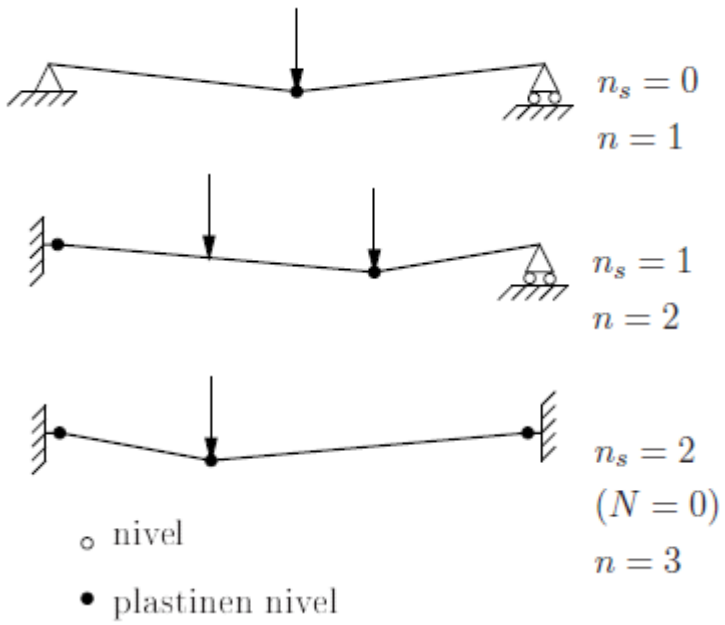
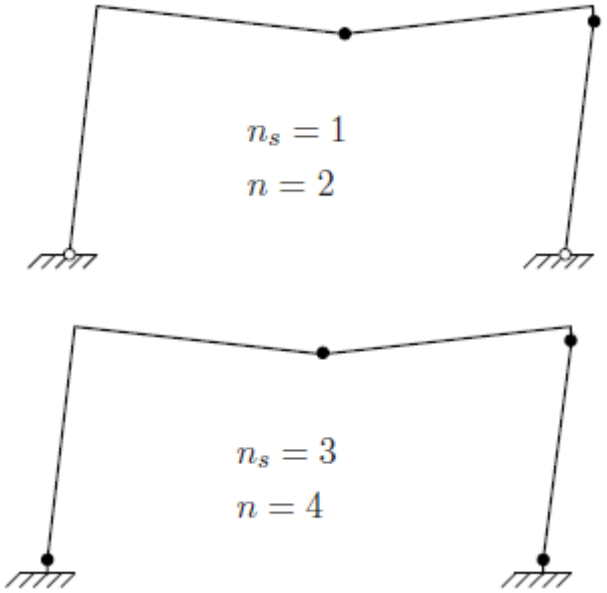
33001 Rakenteiden plastisuusmallit

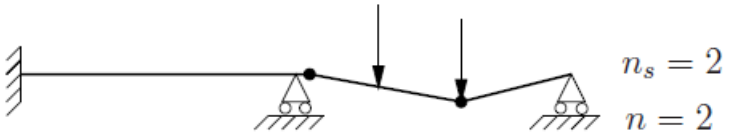
Mekanismimenetelmä

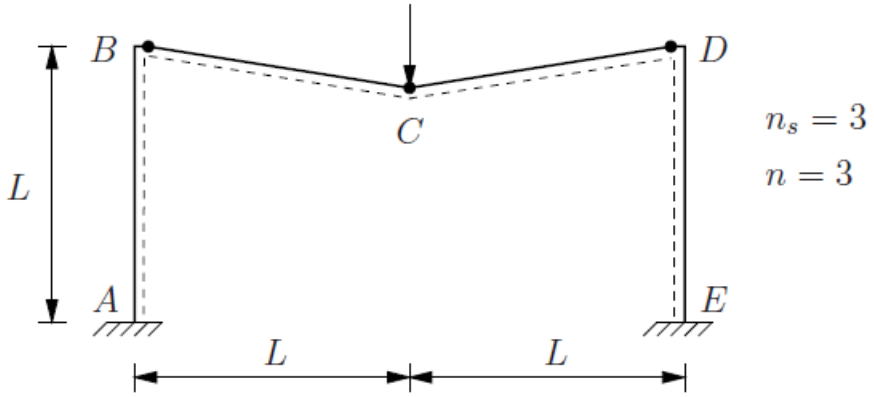
Plastisten nivelten lukumäärä n: $n = n_s + 1$

n_S = staattisen määräämättömyyden kertaluku









Alitäydellisiä mekanismeja.

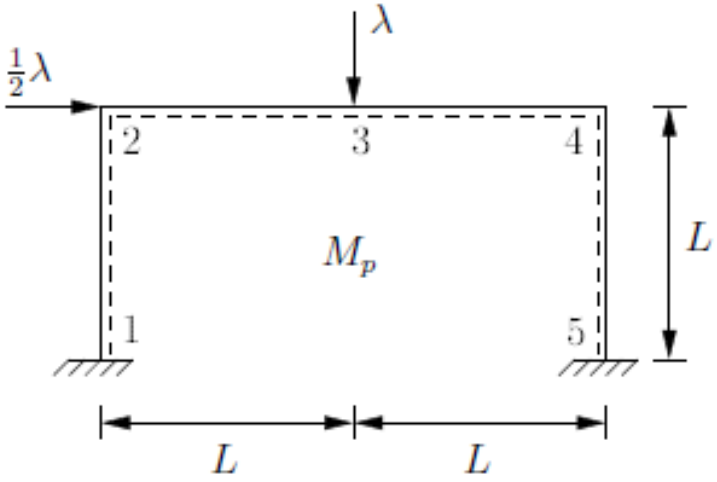
Perusmekanismien lukumäärä m:

$$m = s - n_s$$

s = kriittisten leikkausten lukumäärä

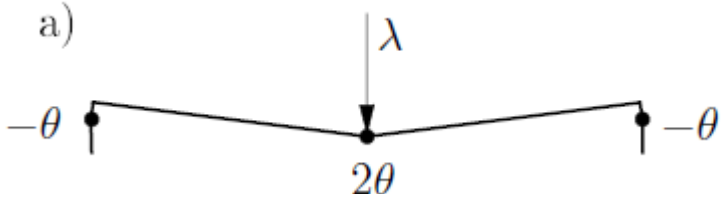
n_s = staattisen määräämättömyyden kertaluku

Esim. 5.1

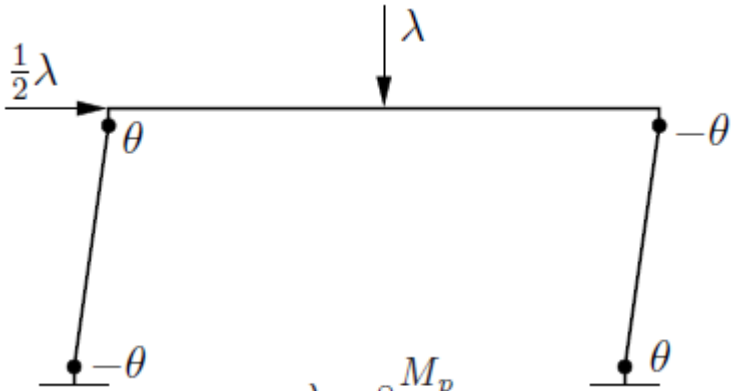


$$m = s - n_s = 5 - 3 = 2$$

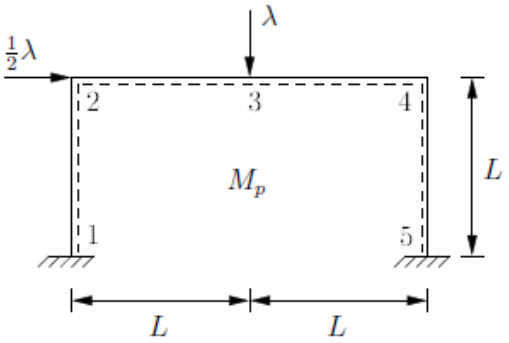
a)

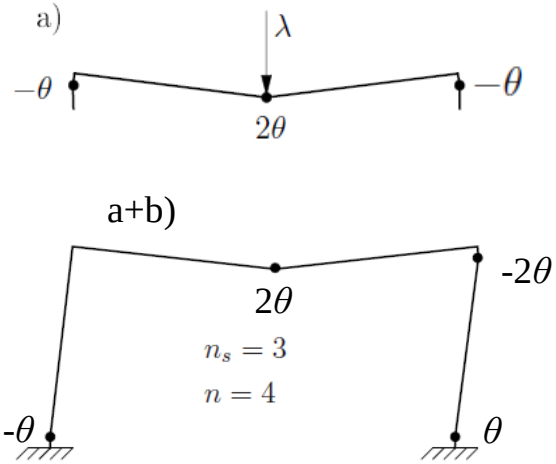


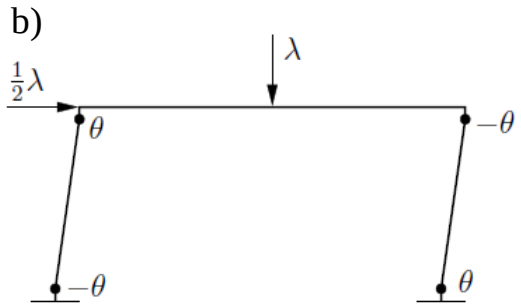
$$\lambda = 4\frac{M_p}{L}.$$



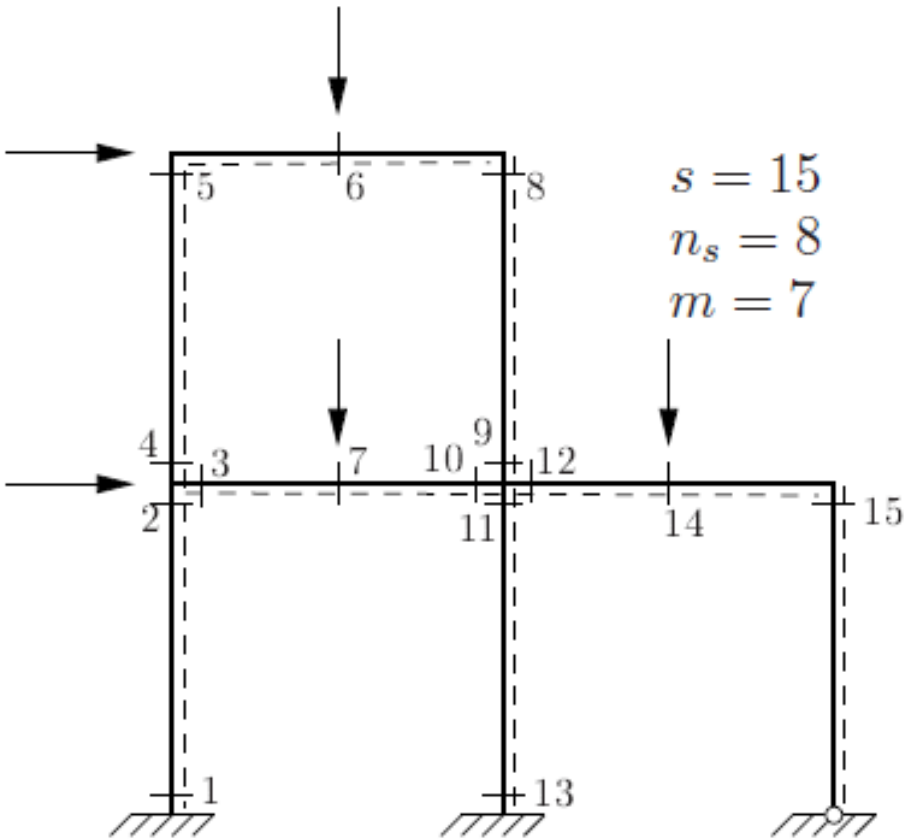
$$\lambda = 8\frac{M_p}{L}.$$







	1	2	3	4	5	$\sum \lvert M_{pj}\theta_j \rvert$	$\sum F_i\delta_i$	λ
a		$-\theta$	2θ	$-\theta$		$4M_p\theta$	$\lambda L\theta$	$4M_p/L$
b	$-\theta$	θ		$-\theta$	θ	$4M_p\theta$	$\frac{1}{2}\lambda L\theta$	$8M_p/L$
$a+b$	$-\theta$	0	2θ	-2θ	θ	$6M_p\theta$	$\frac{3}{2}\lambda L\theta$	$4M_p/L$
$2a+b$	$-\theta$	$-\theta$	4θ	-3θ	θ	$10M_p\theta$	$\frac{5}{2}\lambda L\theta$	$4M_p/L$



$s = 15$

$n_s = 8$

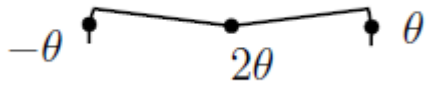
$m = 7$

s = kriittisten leikkausten lukumäärä

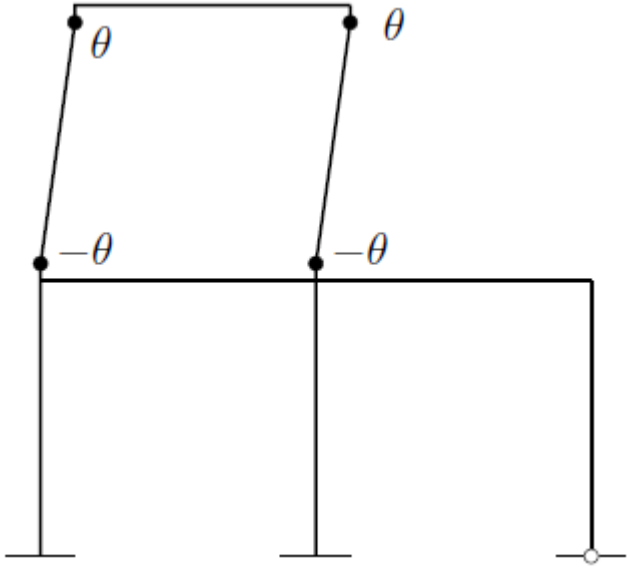
n_s = staattisen määräämättömyyden kertaluku

$m = s - n_s$ perusmekanismien lukumäärä

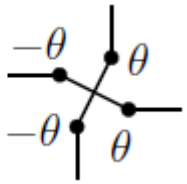
a)

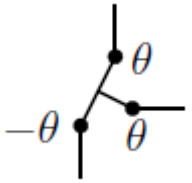

$-\theta$
2θ
θ

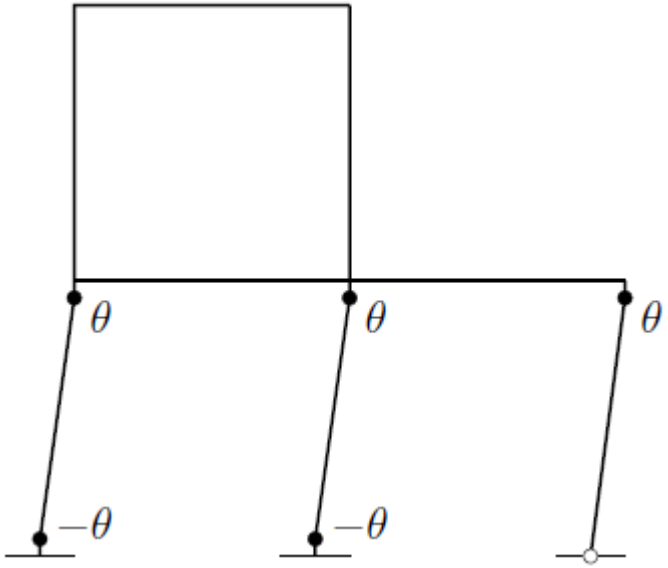
b)


θ
θ
$-\theta$
$-\theta$

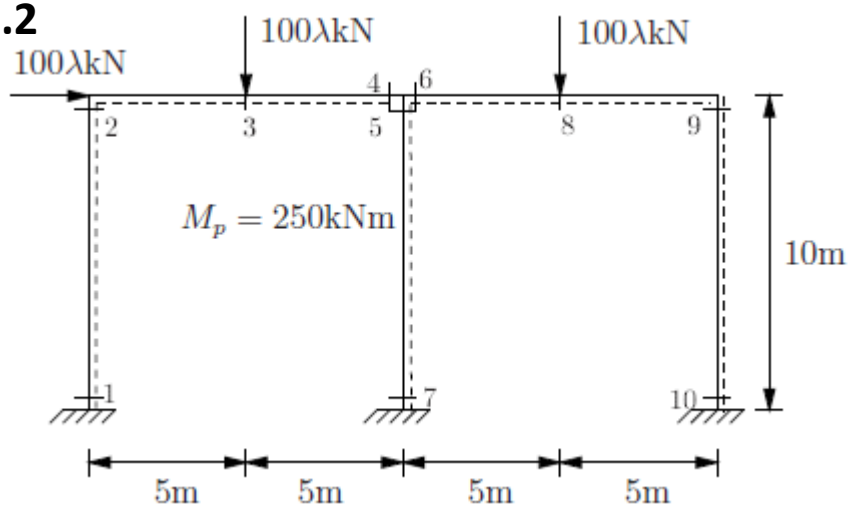
c)


$-\theta$
θ
$-\theta$
θ

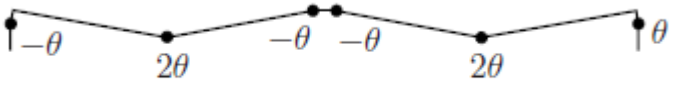

θ
$-\theta$
θ


θ
θ
θ
$-\theta$
$-\theta$

Esim. 5.2

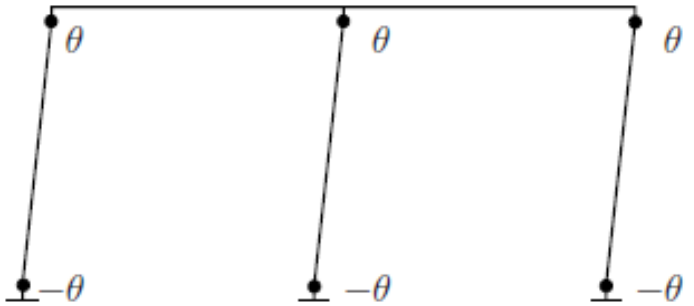


$m = s - n_s = 10 - 6 = 4.$

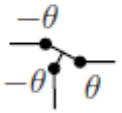


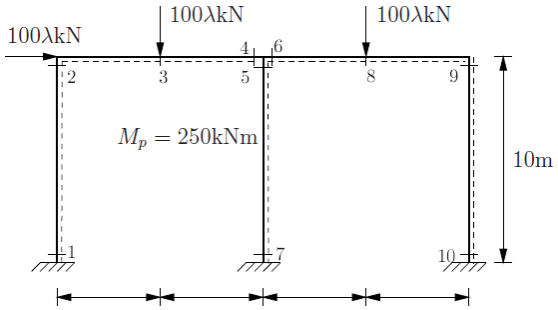
a) b)

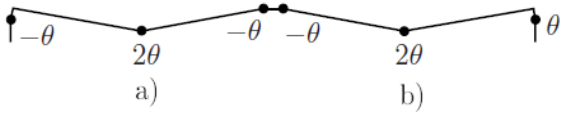
c)



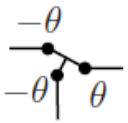
d)

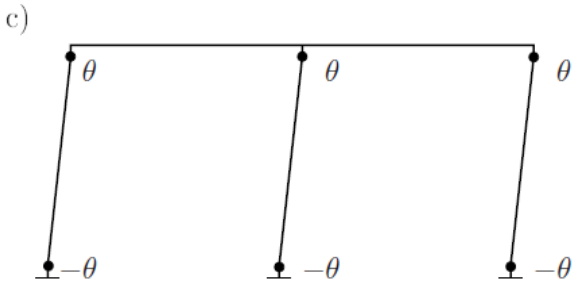






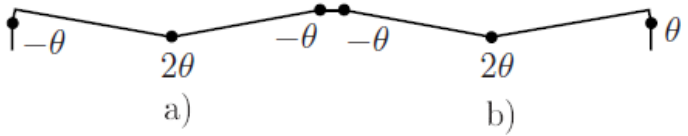
a) b)





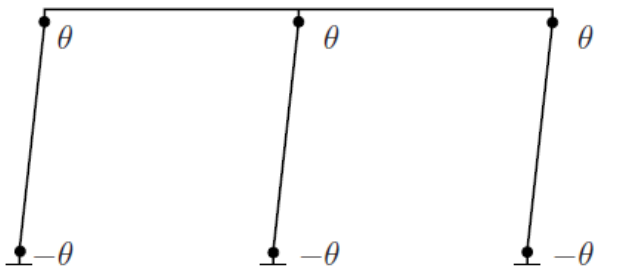
M_{pj}	1 M_p	2 M_p	3 M_p	4 M_p	5 M_p	6 M_p	7 M_p	8 M_p	9 M_p	10 M_p	$\sum \lvert M_{pj}\theta_j \rvert$ [MNm]	$\sum F_i\delta_i$ [MNm]	λ
a		-1	2	-1							1.0θ	$0.5\lambda\theta$	2
b						-1		2	1		1.0θ	$0.5\lambda\theta$	2
c	-1	1			1		-1		1	-1	1.5θ	$1.0\lambda\theta$	1.5
d				-1	-1	1						–	–
e	-1	0	2	-1	1		-1		1	-1	2.0θ	$1.5\lambda\theta$	1.333
f	-1	1			1	-1	-1	2	2	-1	2.5θ	$1.5\lambda\theta$	1.666
g	-1	1		-1	0	0	-1	2	2	-1	2.25θ	$1.5\lambda\theta$	1.5
h	-1	0	2	-2			-1	2	2	-1	2.75θ	$2.0\lambda\theta$	1.375

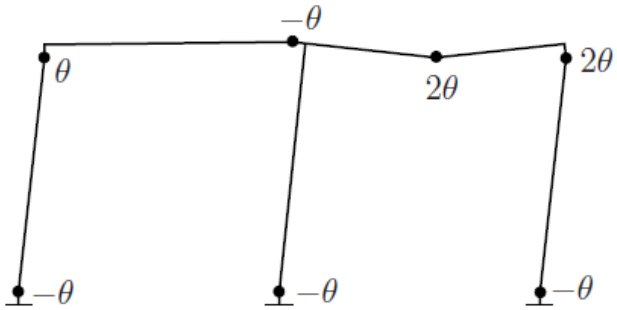
$e = a + c$,
$f = b + c$,
$g = f + d$
$h = g + a$



a) b)

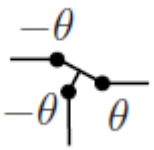
c)

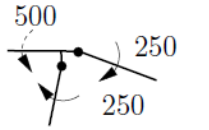


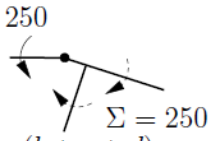


Mekanismi $((b+c)+d)$.

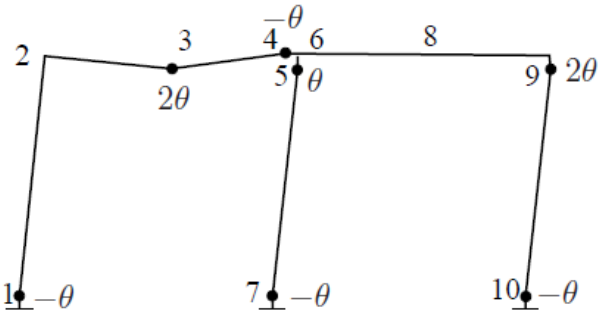
d)







$(b+c+d)$

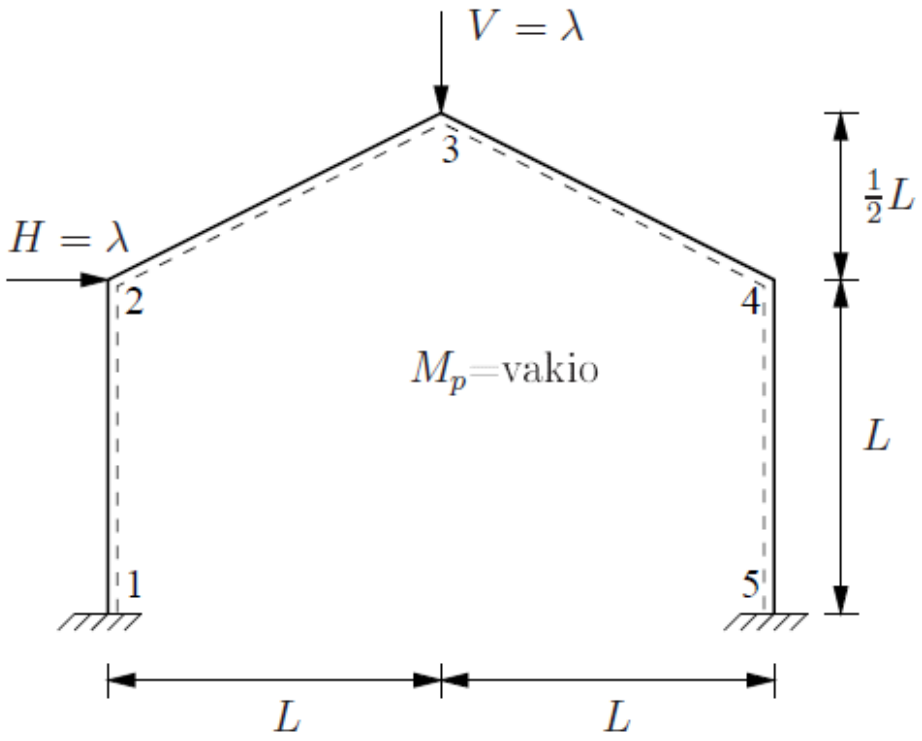


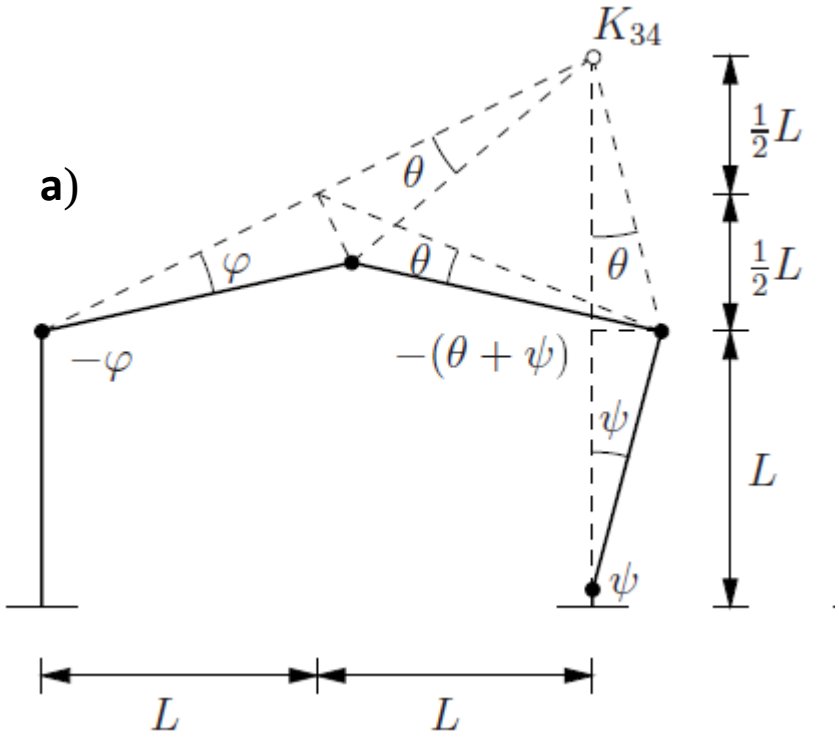
Mekanismi $(a+c)$.

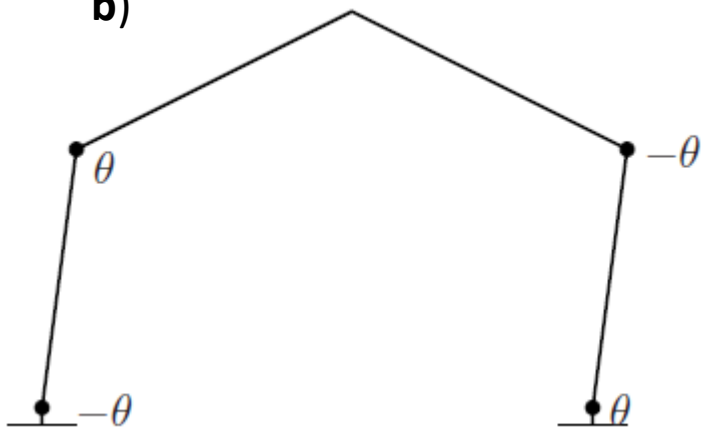
33001 Rakenteiden plastisuusmallit

Mekanismimenetelmä b

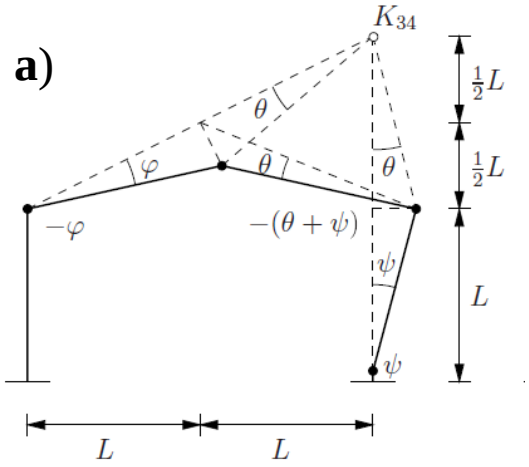
Esim. 5.3




a)
K_{34}
$\frac{1}{2}L$
$\frac{1}{2}L$
L
φ
θ
θ
θ
$-\varphi$
$-(\theta+\psi)$
ψ
ψ
L
L

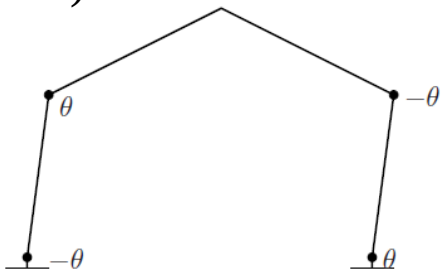

b)
θ
$-\theta$
$-\theta$
θ

a)

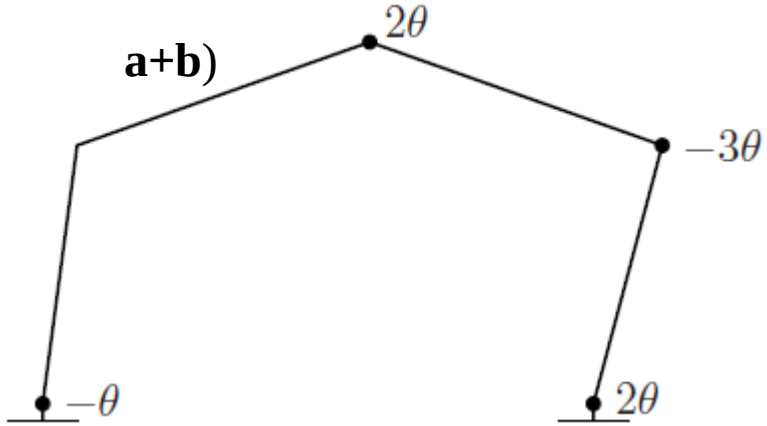


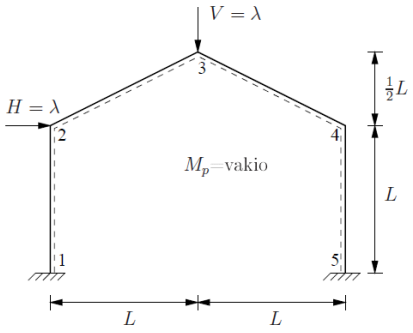
$\varphi = \theta$ ja $\psi = \theta$.

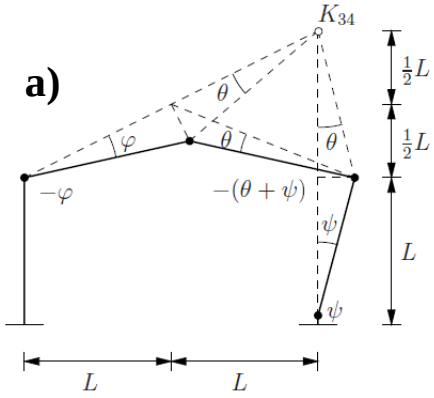
b)

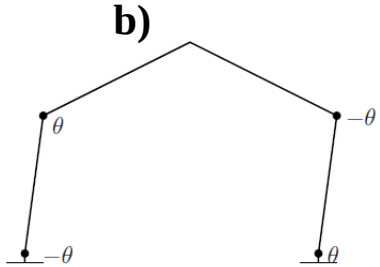


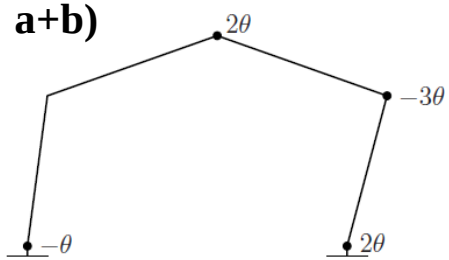
a+b)





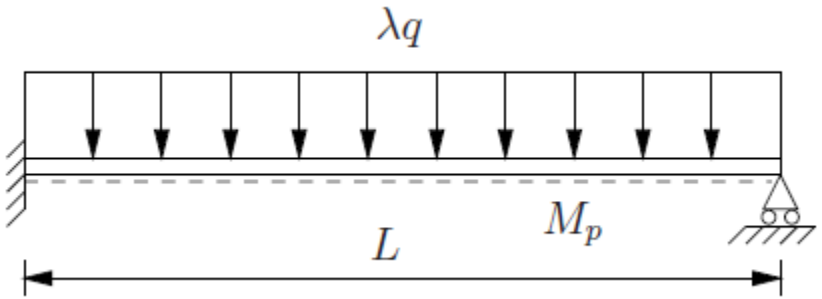


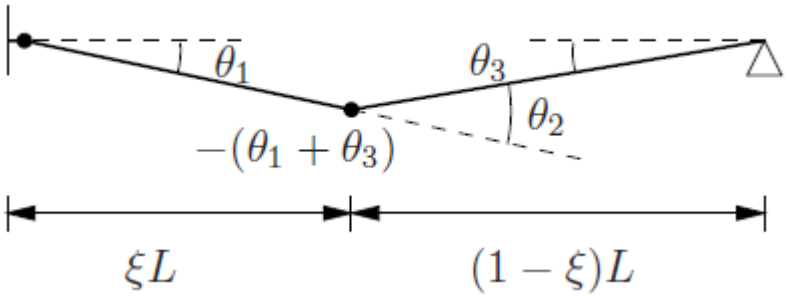




	1	2	3	4	5	$\sum \lvert M_p\theta \rvert$	$\sum F\delta$	λ
a		$-\theta$	2θ	-2θ	θ	$6M_p\theta$	$\lambda L\theta$	$6M_p/L$
b	$-\theta$	θ		$-\theta$	θ	$4M_p\theta$	$\lambda L\theta$	$4M_p/L$
$a+b$	$-\theta$	0	2θ	-3θ	2θ	$8M_p\theta$	$2\lambda L\theta$	$4M_p/L$

Esim. 5.4





$$\theta_1 \equiv \theta.$$

$$\theta_2 = \frac{1}{1-\xi}\theta.$$

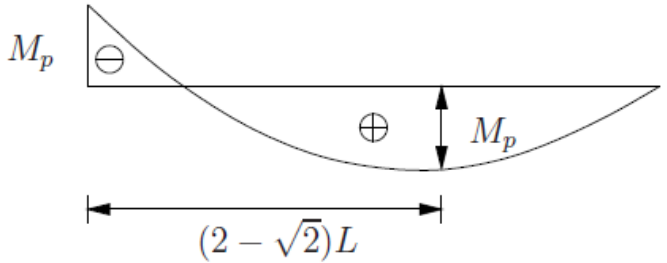
$$\theta M_p + \frac{1}{1-\xi}\theta M_p = \lambda q \cdot \xi L\theta \cdot \frac{L}{2}$$

$$\lambda q = \frac{2(2-\xi)}{\xi(1-\xi)}\frac{M_p}{L^2}$$

$$\frac{d(\lambda q)}{d\xi} = 0, \qquad \frac{(-2)\xi(1-\xi)-(4-2\xi)(1-2\xi)}{\xi^2(1-\xi)^2}\frac{M_p}{L^2} = 0$$

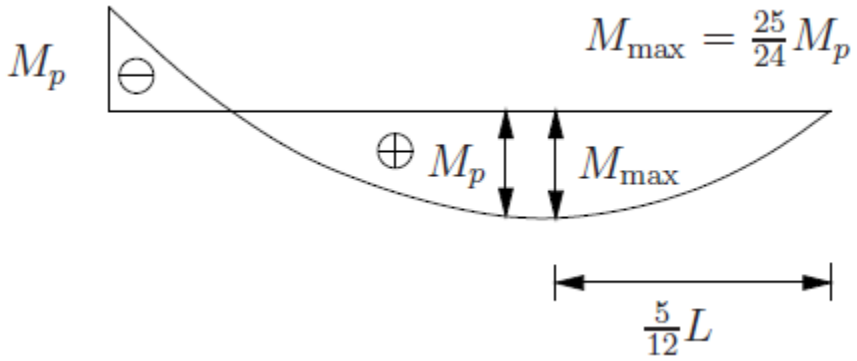
$$\xi^2 - 4\xi + 2 = 0 \quad \Rightarrow \quad \xi_{1,2} = 2 \pm \sqrt{4-2} \qquad \xi_2 = 2 - \sqrt{2}$$

$$(\lambda q)_p = (6+4\sqrt{2})\frac{M_p}{L^2} \approx 11.66\frac{M_p}{L^2}$$



Likimääräinen tapa: oletetaan nivel palkin keskelle

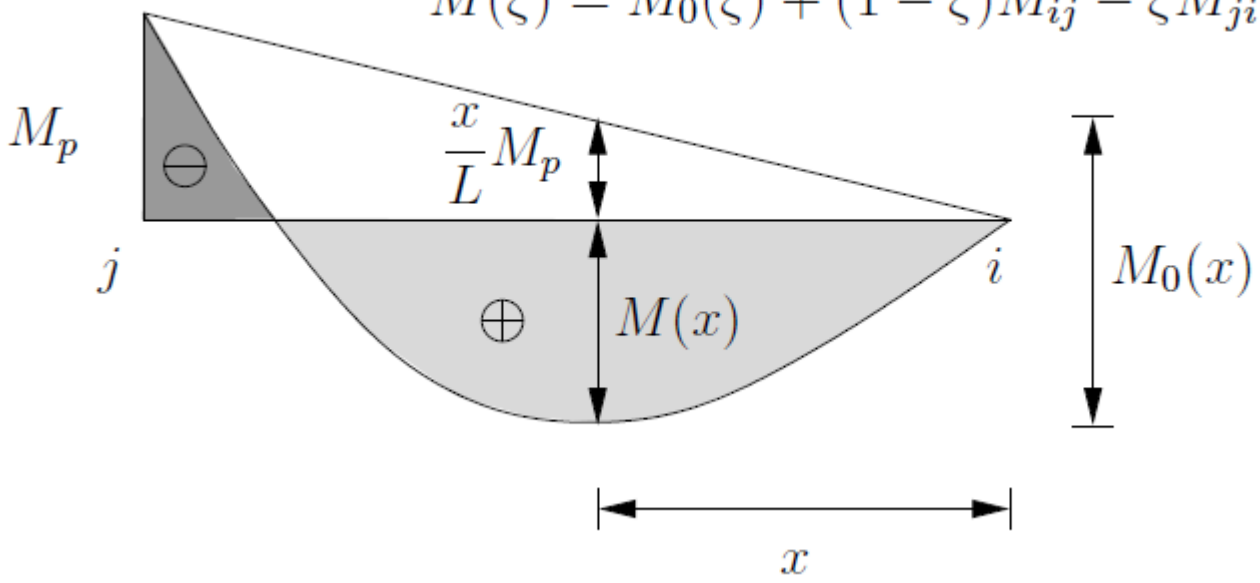
$$3M_p\theta = \lambda q \frac{L}{2} \cdot \frac{L}{2}\theta \quad \Rightarrow \quad \lambda = \frac{12M_p}{qL^2}.$$

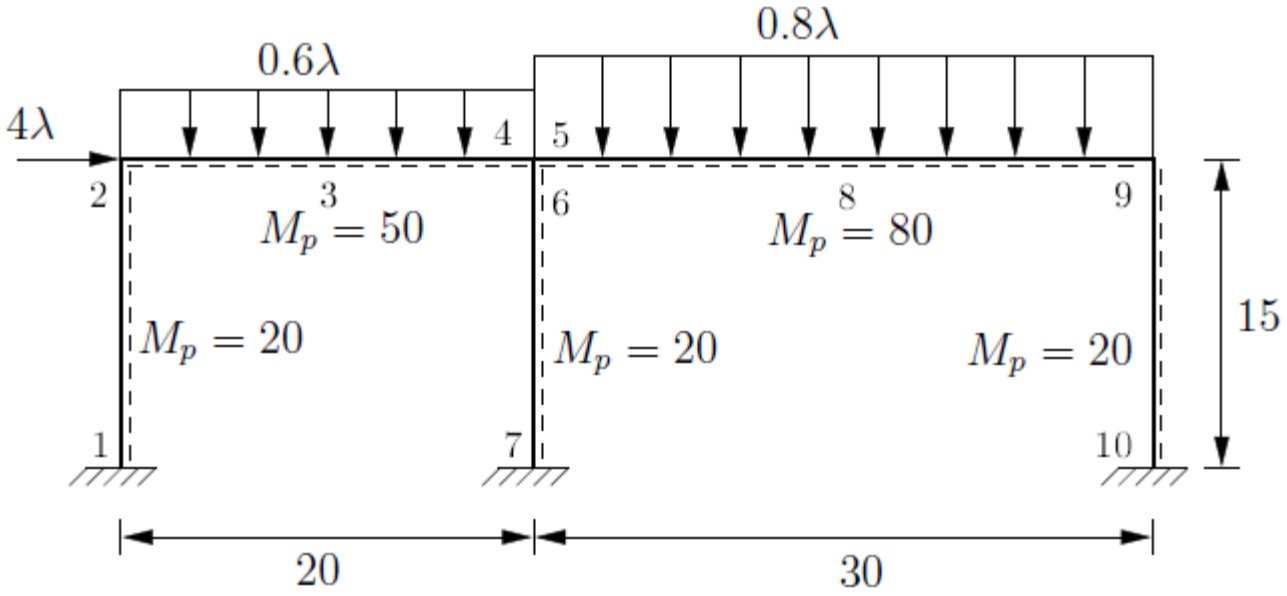


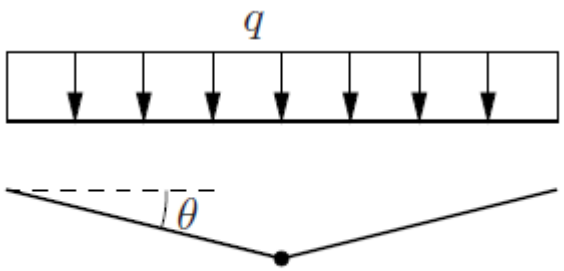
$$\lambda_p = \frac{24}{25} \cdot 12\frac{M_p}{qL^2} \approx 11.52\frac{M_p}{qL^2}$$

$$11.52\frac{M_p}{qL^2} < \lambda_p < 12\frac{M_p}{qL^2}$$

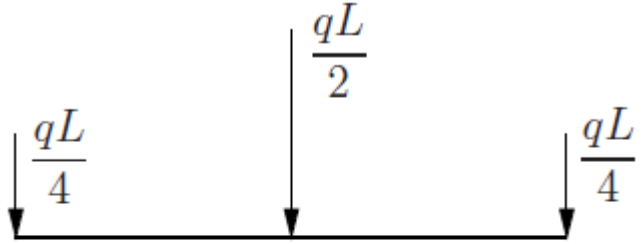
$$M(\xi) = M_0(\xi) + (1-\xi)M_{ij} - \xi M_{ji},\ \xi = x/L$$



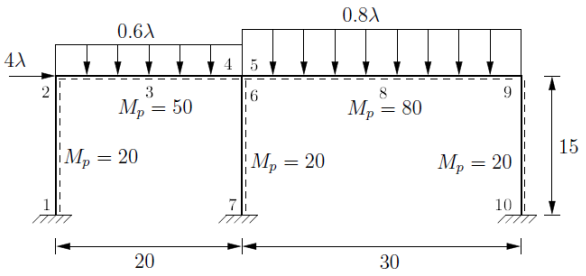

0.8λ
0.6λ
4λ
4
5
2
3
6
8
9
$M_p = 50$
$M_p = 80$
$M_p = 20$
$M_p = 20$
$M_p = 20$
15
1
7
10
20
30

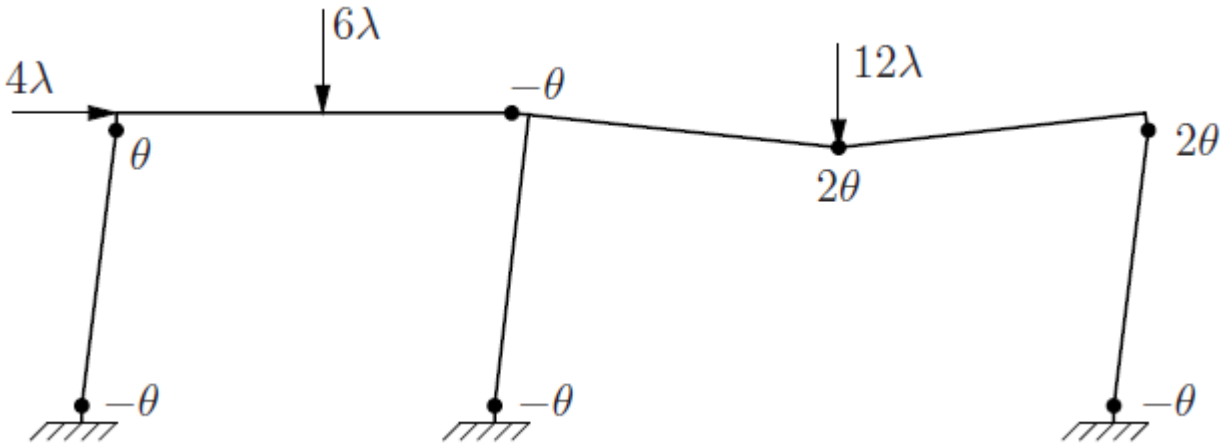


$$W_u = (qL^2/4)\theta$$



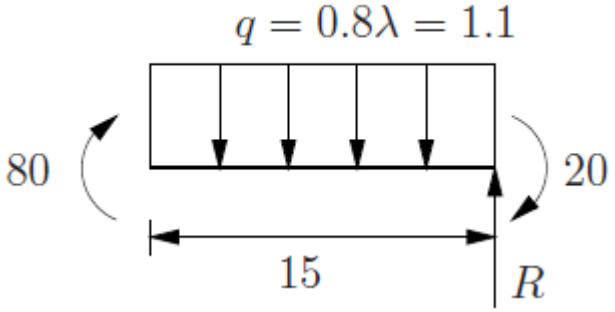
$$W_u = (qL^2/4)\theta$$

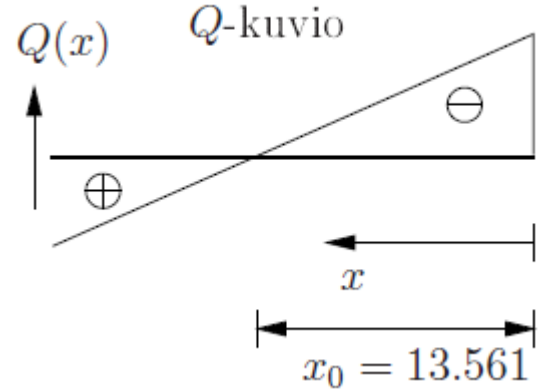


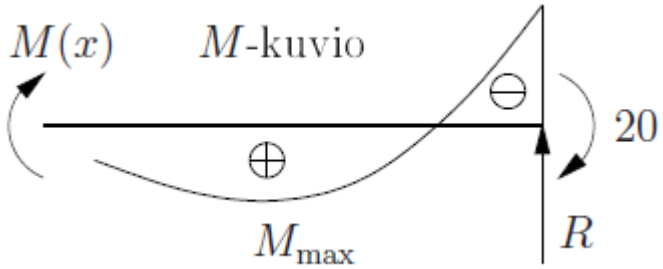


$$20 \cdot 6\theta + 50 \cdot \theta + 80 \cdot 2\theta = 4\lambda \cdot \theta \cdot 15 + 12\lambda \cdot \theta \cdot 15 = 240\lambda\theta$$

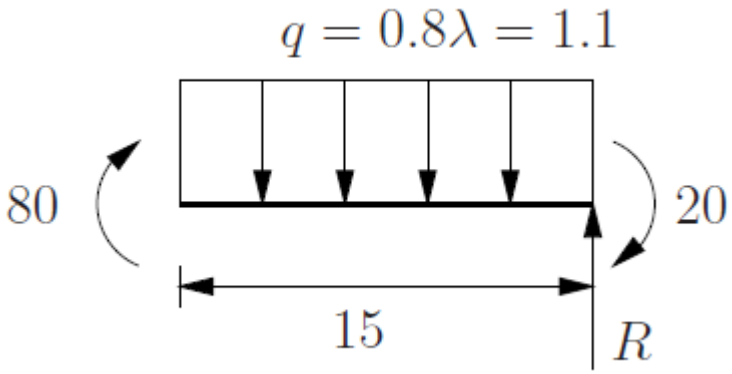
$$\lambda = \frac{330}{240} = \frac{11}{8} = 1.375$$







$$R \cdot 15 - 80 - 20 - 1.1 \cdot 15 \cdot \frac{15}{2} = 0 \quad \Rightarrow \quad R \approx 14.9166$$



$$Q(x) = -R + q \cdot x = -14.917 + 1.1x$$

$$Q(x) = 0, \text{ kun } x_0 \approx 13.561$$

$$M(x) = R \cdot x - qx^2/2 - 20$$

$$M(x_0) = M_{\max} = 81.14 > M_p = 80$$

$$\lambda^- = (80/81.14)1.375 = 1.356$$

$$1.356 \leq \lambda_p \leq 1.375$$