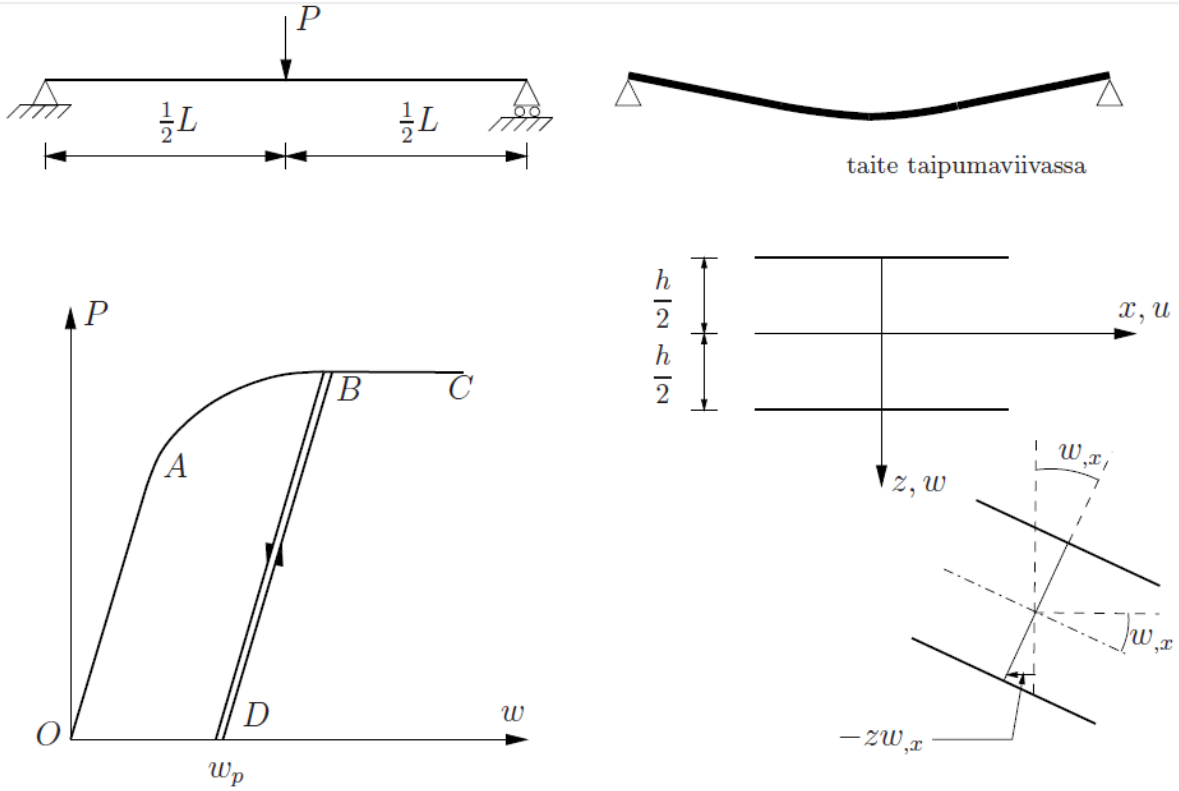
Luento 2:

33001 Rakenteiden plastisuusmallit

Palkin plastinen taivutus



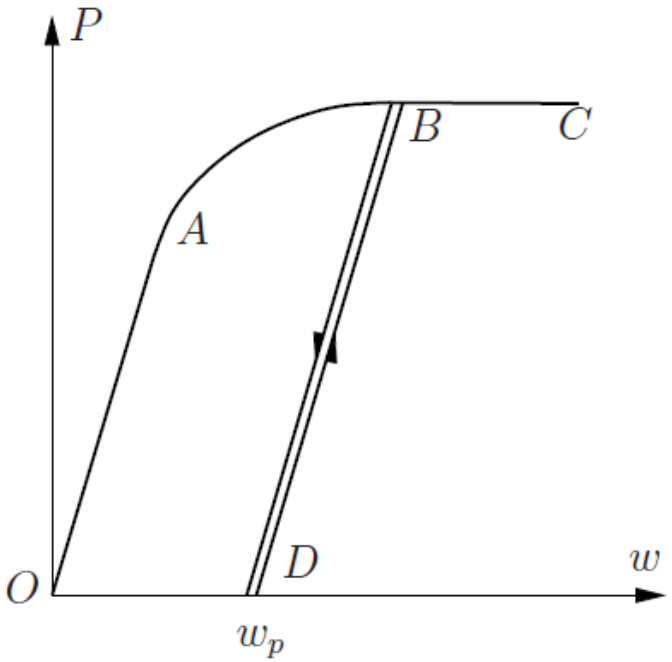
Kuva 2.1 Palkin plastinen taivutus.

$$u = -z\frac{\partial w}{\partial x} \quad , \quad \varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon = \varepsilon_x \quad \text{ja} \quad \kappa = -\frac{\partial^2 w}{\partial x^2} \equiv -w_{,xx}$$

$$\sigma = E\varepsilon \ , \quad \text{kun} \quad |\varepsilon| \leq \frac{\sigma_m}{E}$$

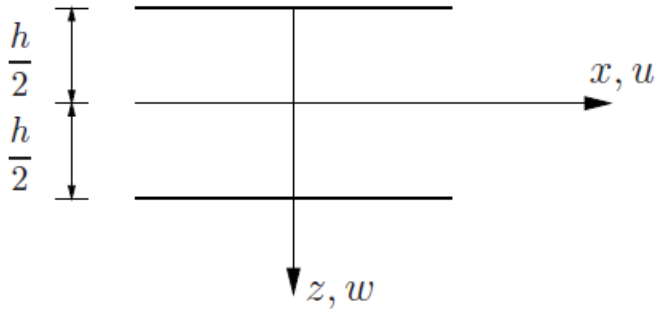
$$\sigma = \text{sgn}(\varepsilon)\sigma_m \ , \quad \text{kun} \quad |\varepsilon| > \frac{\sigma_m}{E}$$



kimmoinen

$$M = \int_{-h/2}^{h/2} z\sigma b\, dz = \int_{-h/2}^{h/2} z\frac{2z}{h}\sigma_m b\, dz = \frac{bh^2}{6}\sigma_m$$

$$M = W\sigma_m = M_m$$



plastinen

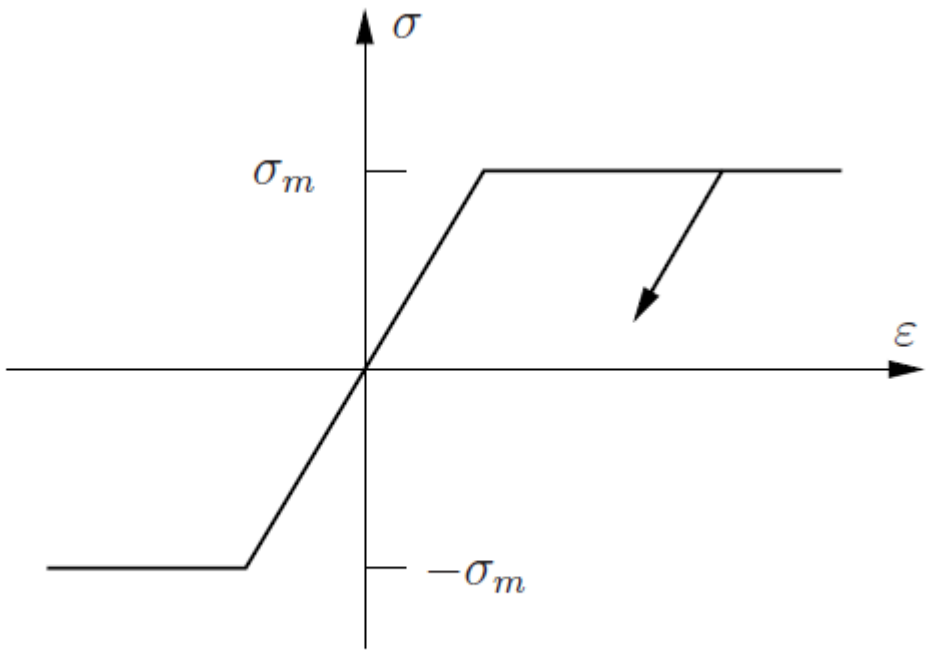
$$M = \int_A z\sigma\, dA = (-\sigma_m)\int_{-h/2}^{0} z\, dA + \sigma_m \int_0^{h/2} z\, dA = 2\sigma_m \int_0^{h/2} zb\, dz = \sigma_m \frac{bh^2}{4}$$

$$M = M_p = \sigma_m W_p$$

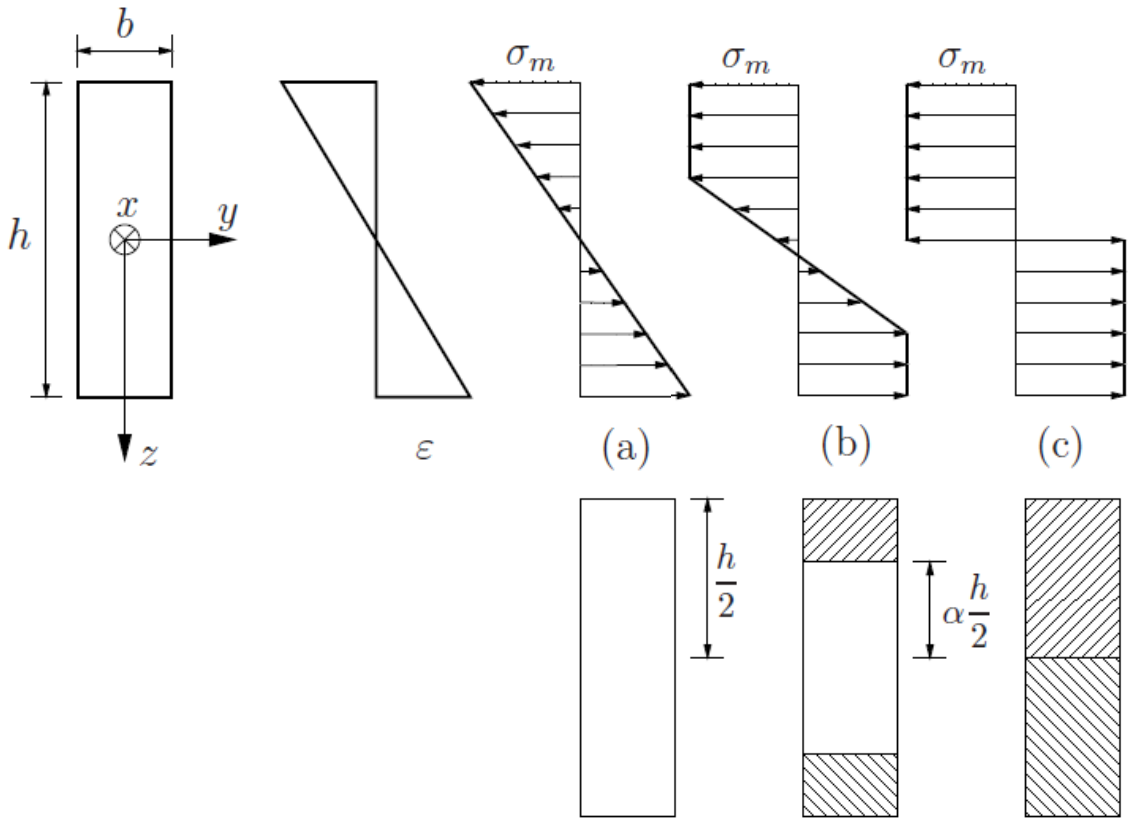
$$\Phi = \frac{W_p}{W}$$

Suorakaidepoikkileikkaus:

$$\Phi = \frac{bh^2/4}{bh^2/6} = \frac{3}{2}$$



Kimmoinen ideaaliplastinen materiaali.


b
h
x
y
z
ε
σm
σm
σm
(a)
(b)
(c)
h/2
αh/2

$$M_p = \sigma_m A_1 z_1 + \sigma_m A_2 z_2 = (A_1 z_1 + A_2 z_2)\sigma_m = W_p \sigma_m$$

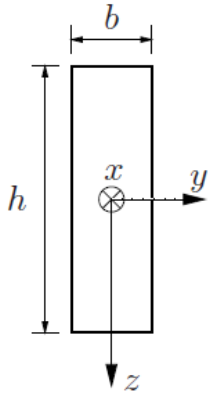
$$W_p = z_1 A_1 + z_2 A_2$$

suorakaidepoikkileikkaus:

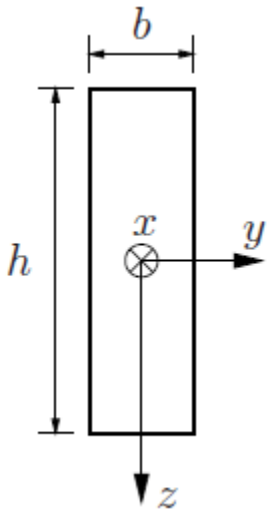
$$A_1 = A_2 = bh/2,$$

$$z_1 = z_2 = h/4$$

$$W_p = bh^2/4.$$



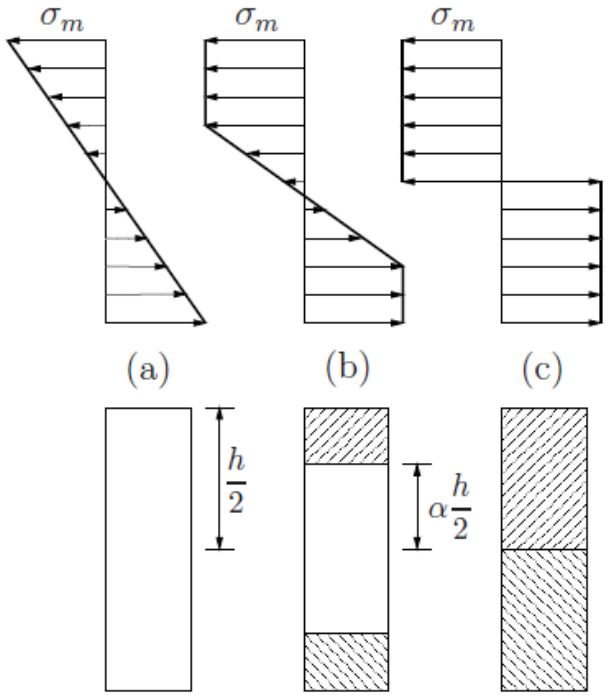
taivutusmomentti: $M = \int_A z\sigma \, dA$

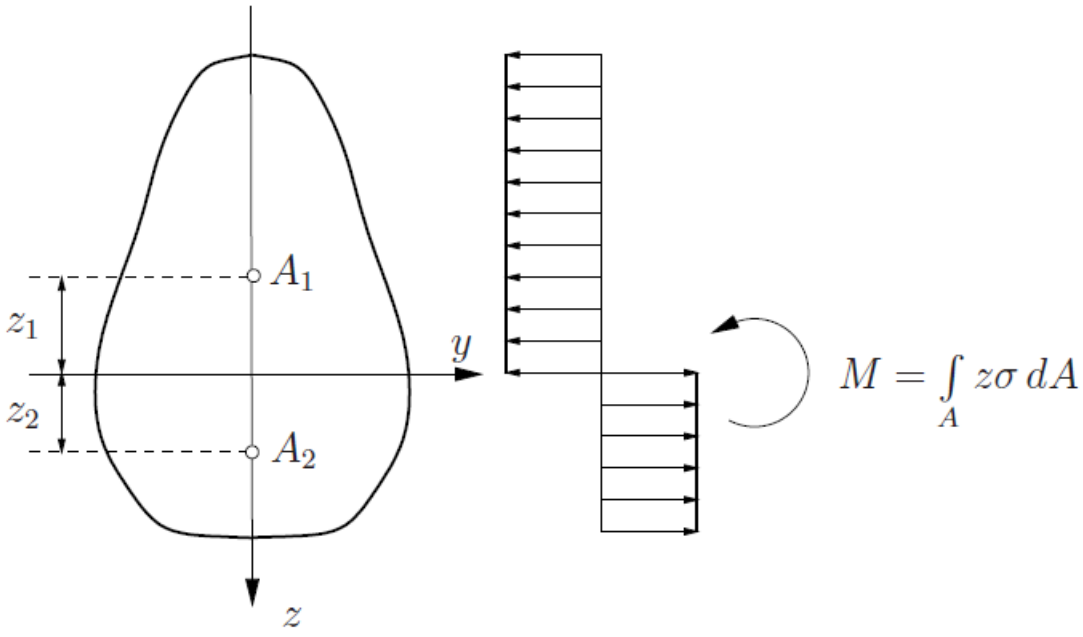


$$M = 2\int_0^{\alpha h/2} z\sigma b \, dz + 2\int_{\alpha h/2}^{h/2z} \sigma_m b \, dz$$

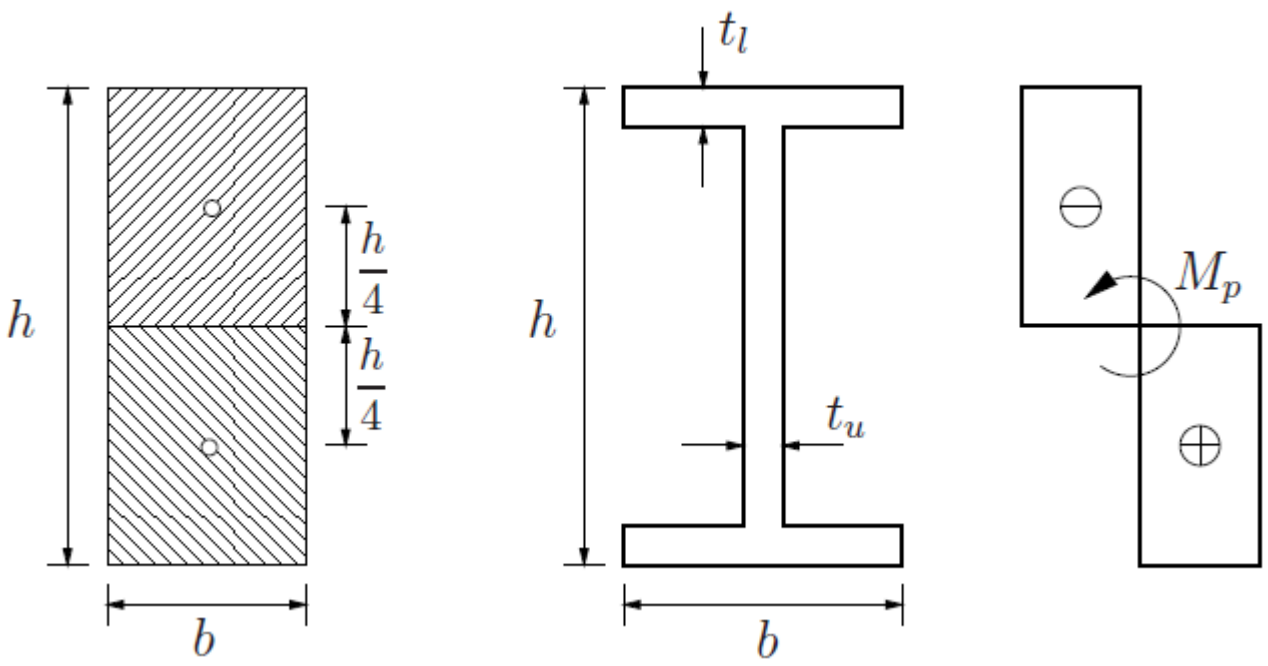
$$M = 2\int_0^{\alpha h/2} z\frac{z}{\alpha h/2}\sigma_m b \, dz + 2\sigma_m b \int_{\alpha h/2}^{h/2} z \, dz$$

$$M = \frac{bh^2}{6}\sigma_m \left(\frac{3}{2} - \frac{1}{2}\alpha^2\right)$$





Symmetrinen poikkileikkaus.

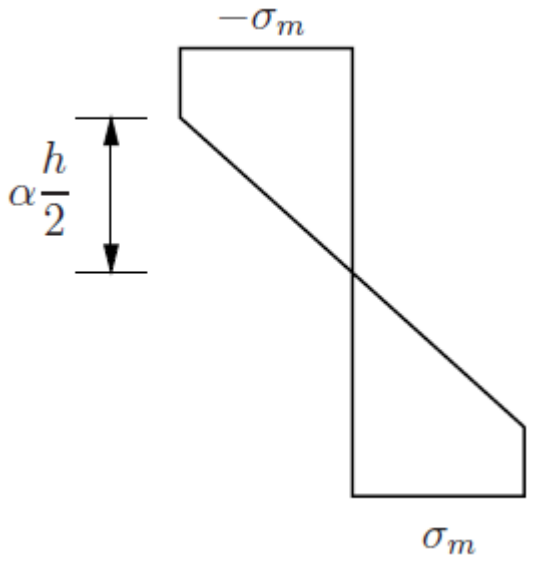


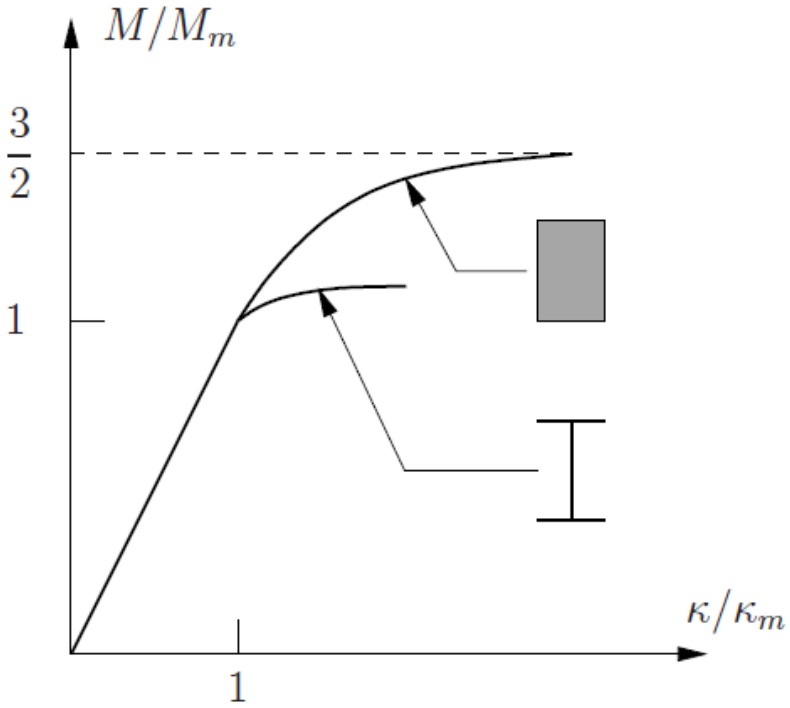
Suorakaide- ja I-poikkileikkaus.

käyristyminen: $\kappa(a) = \frac{\varepsilon_m}{h/2} = \kappa_m \Longrightarrow \kappa(x) = \frac{x}{a}\kappa_m$

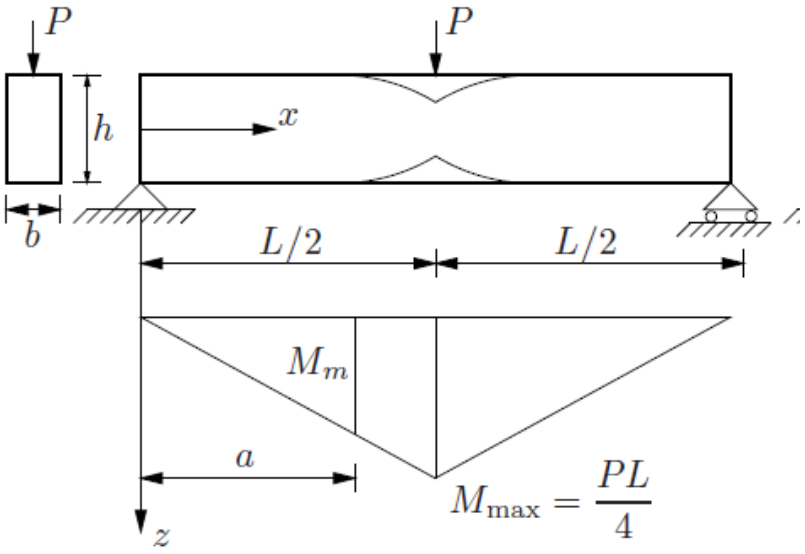
$$\frac{\varepsilon}{\varepsilon_m} = \frac{\kappa}{\kappa_m} = \frac{h/2}{\alpha(h/2)} = \frac{1}{\alpha}$$

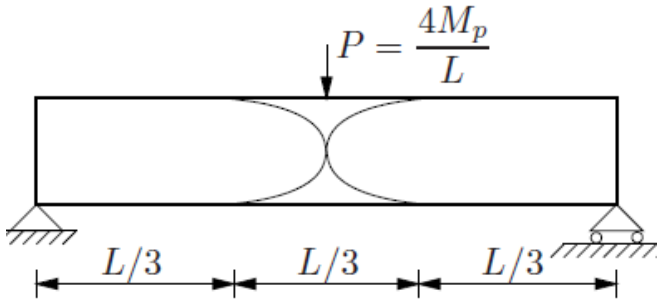
$$\Phi = \frac{M_p}{M_m} = \frac{W_p}{W}$$

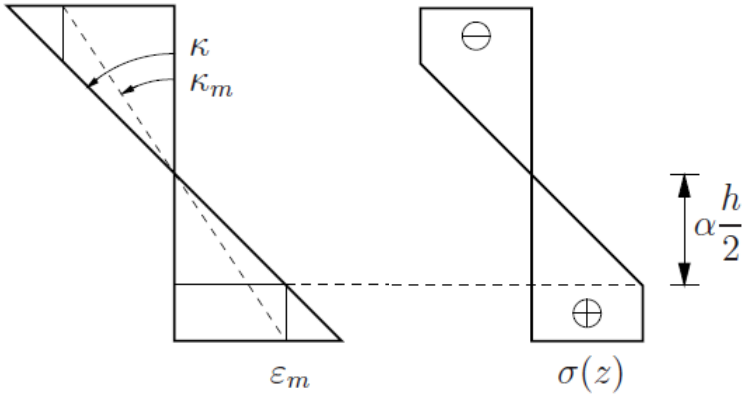


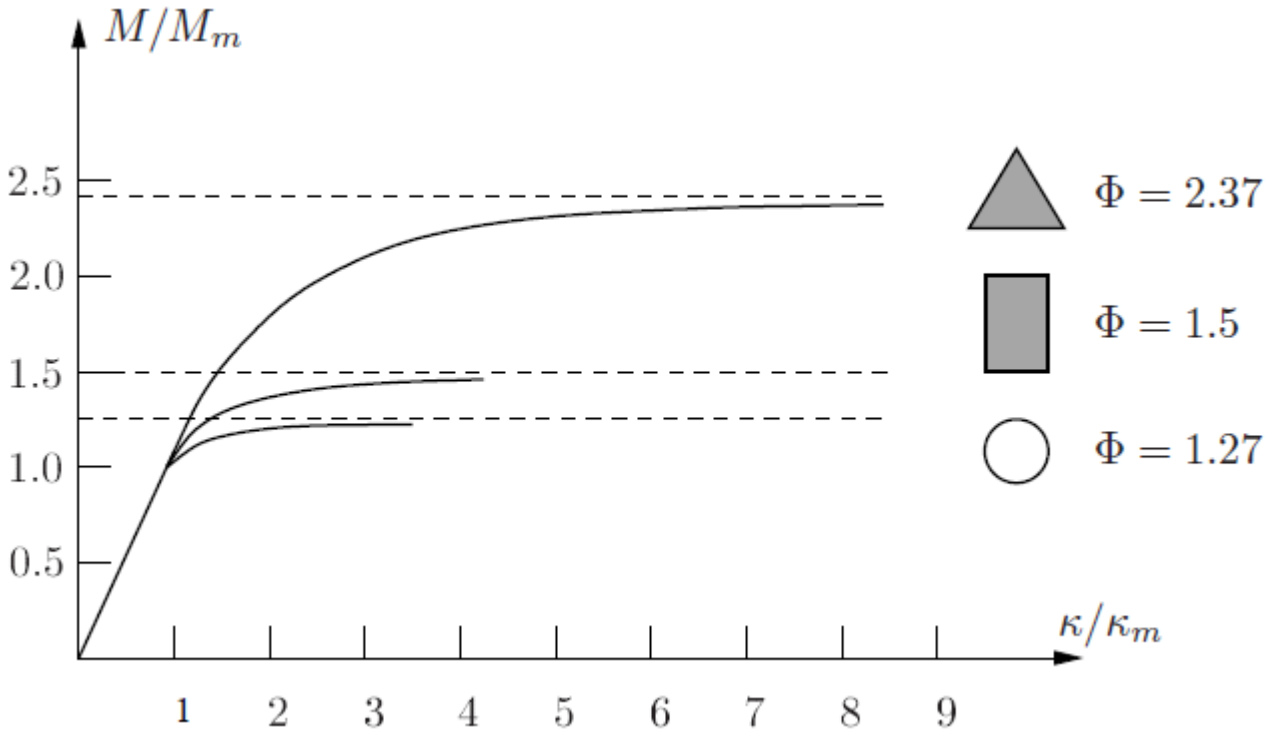


Suorakaide- ja I-poikkileikkauksen momentin ja käyristymän välinen riippuvaisuus.

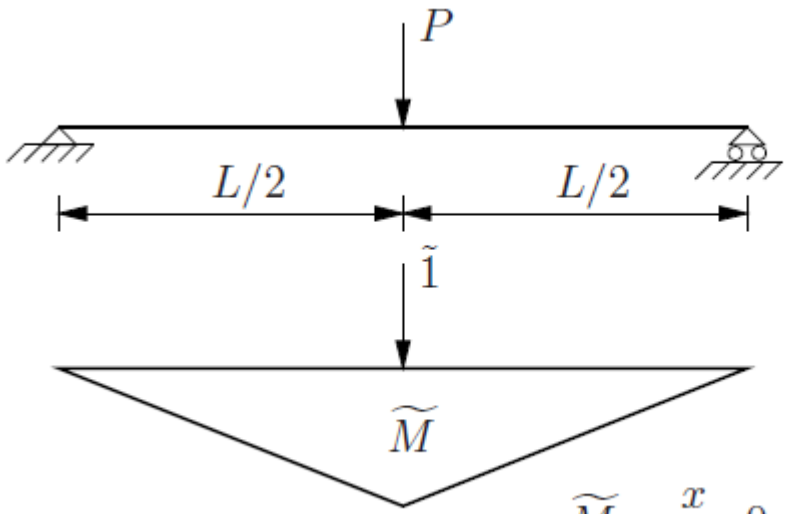

P
P
h
x
b
L/2
L/2
M_m
a
z
M_max = PL/4


P = 4M_p/L
L/3
L/3
L/3


κ
κ_m
α h/2
ε_m
σ(z)


M/M_m
2.5
2.0
1.5
1.0
0.5
κ/κ_m
1
2
3
4
5
6
7
8
9
$\Phi = 2.37$
$\Phi = 1.5$
$\Phi = 1.27$

ykkösvoimaperiaate: $\tilde{1}w = \int_0^L \widetilde{M}\kappa dx,$ $\widetilde{M} = \frac{x}{2},\ 0 < x < \frac{L}{2}$



$\widetilde{M} = \frac{x}{2},\ 0 \leq x \leq L/2$

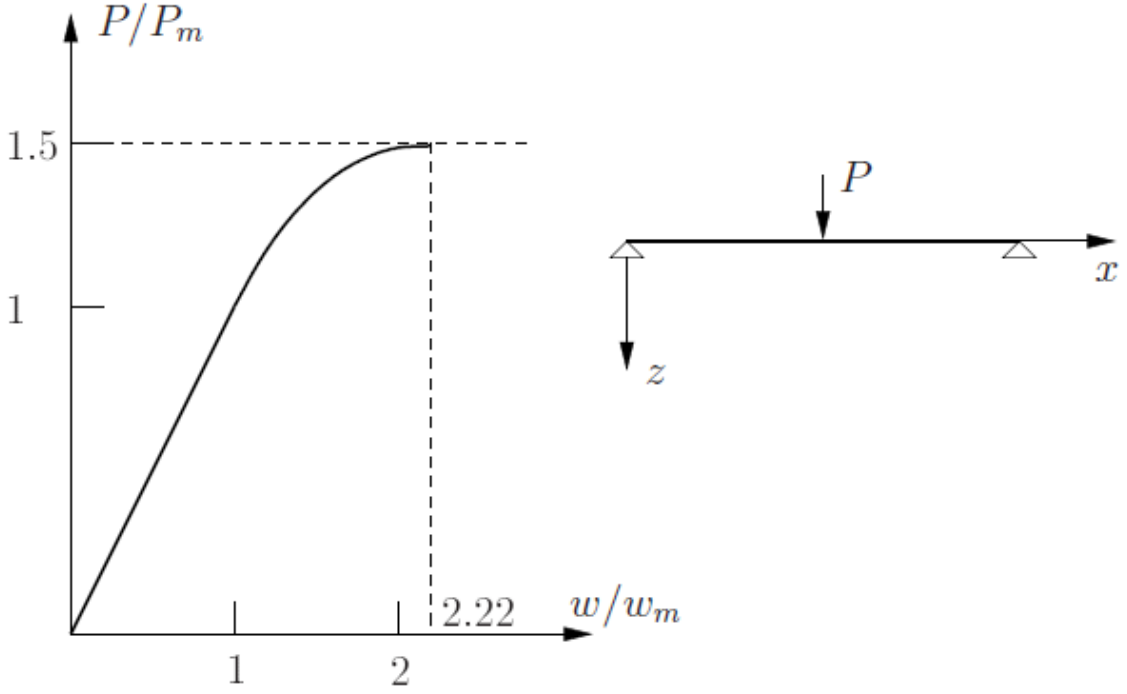
suorakaidepoikkileikkaus:

$$\frac{M(x)}{M_m} = \frac{x}{a} = \frac{1}{2}\left[3 - \left(\frac{\kappa_m}{\kappa}\right)^2\right]$$

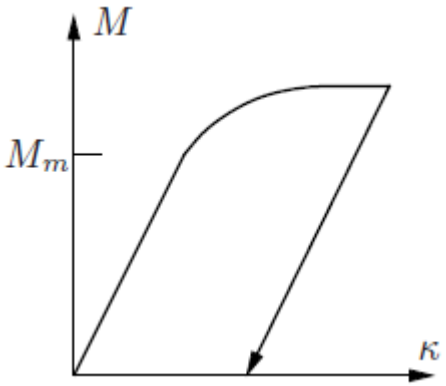
$$\kappa = \frac{\kappa_m}{\sqrt{3 - \dfrac{2x}{a}}}, \quad a < x < \frac{L}{2}$$

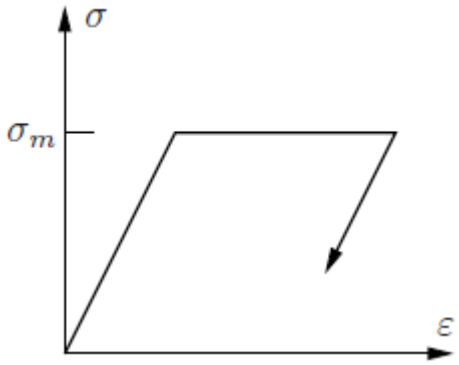
$$w\left(\frac{L}{2}\right) = \int_0^a \kappa_m \frac{x}{a} x dx + \int_a^{L/2} \frac{\kappa_m}{\sqrt{3 - \dfrac{2x}{a}}} x dx,$$

$$w\left(\frac{L}{2}\right) = \left[\frac{5}{3}a^2 - \left(a^2 + a\frac{L}{6}\right)\sqrt{3 - \frac{L}{a}}\right]\kappa_m$$



Vapaasti tuetun palkin kuorma-siirtymäkäyrä

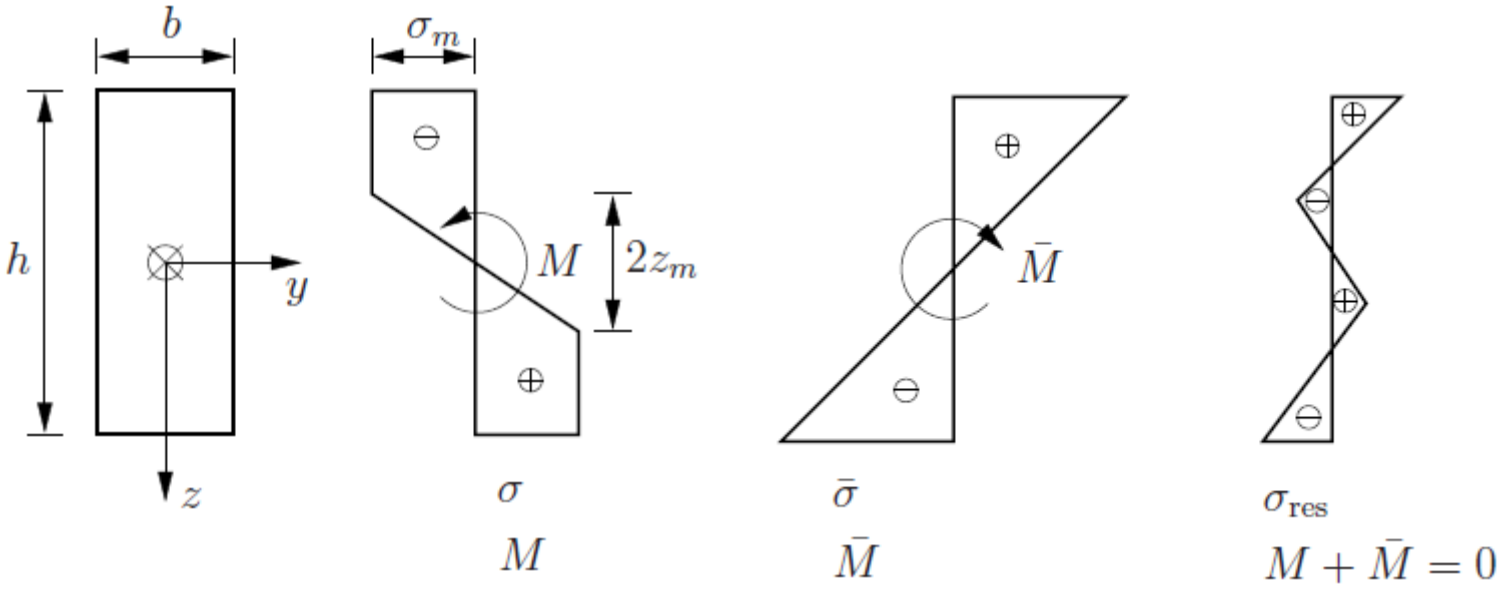




suorakaidepoikkileikkaus:

$$M = \frac{1}{2}\sigma_m W(3-\alpha^2) \qquad \alpha = \frac{2z_m}{h}$$

$$\sigma = \sigma_m \frac{z}{z_m}, \quad |z| \le z_m$$



jäännösjännitys: $\sigma_{\text{res}}(z) = \sigma(z) + \bar{\sigma}(z)$