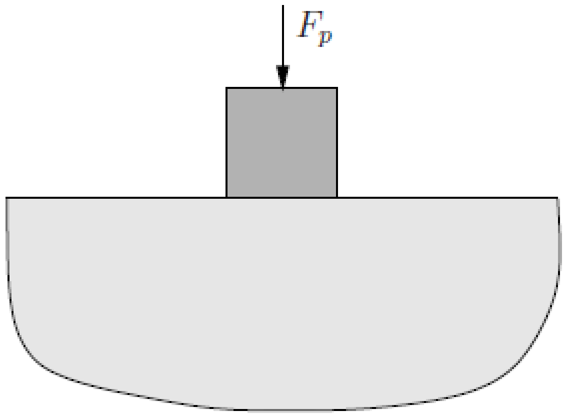
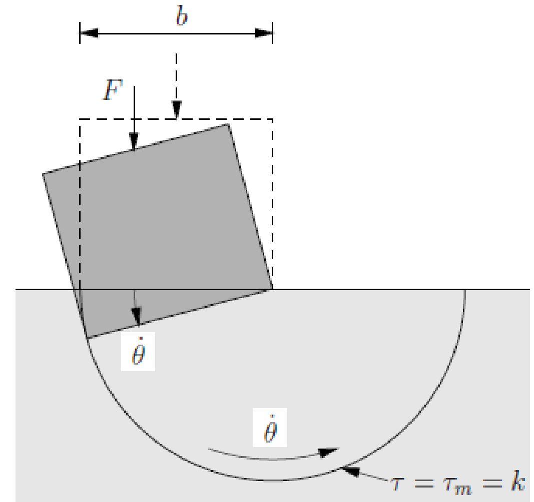
33001 Rakenteiden plastisuusmallit

Tasomuodonmuutostilan ylä- ja alarajaratkaisut



$$\dot{W}_u = \dot{W}_s$$

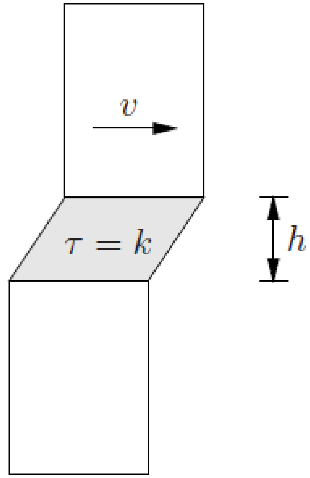


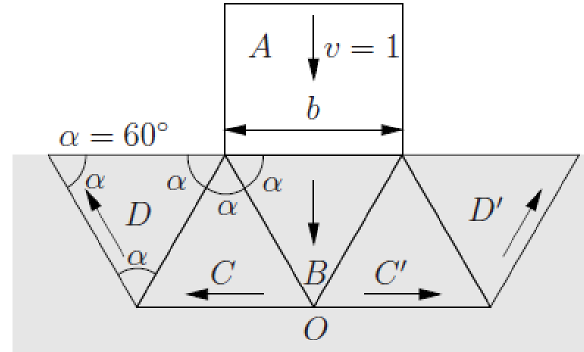
$$\dot{W}_u = \frac{1}{2} F b \dot{\theta}$$

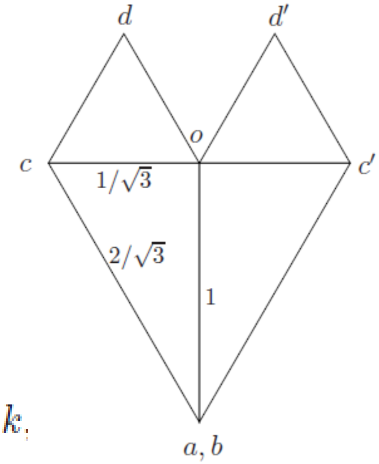
$$\dot{W}_s = \int \tau \dot{\gamma} ds = k b \dot{\theta} \pi b$$

$$F^u = 2\pi k b \approx 6.28 k b$$

Leikkausvyöhyke







$$F \cdot v = 2\{v_{BC} l_{BC} + v_{CO} l_{CO} + v_{CD} l_{CD} + v_{DO} l_{DO}\} k$$

$$\dot{\gamma} = \frac{v}{h}$$

$$\tau = \tau_m \equiv k$$

$$\tau \dot{\gamma} h = k \left[\frac{v}{h}\right] h = kv$$

$$\frac{oc}{oa} = \frac{1}{\sqrt{3}}$$

$$l_{BC} = l_{CO} = \cdots = b$$

$$F^u = 2kb\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) = \frac{10}{\sqrt{3}} kb \approx 5.77kb$$

Jäykkä-plastinen materiaali

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{p} = d\lambda S_{ij}$$

$$S_{ij} = \sigma_{ij} - p\delta_{ij}$$

$$p = \tfrac{1}{3}\sigma_{kk} \equiv \tfrac{1}{3}\sum_{k=1}^{3}\sigma_{kk}$$

$$d\varepsilon_x = \frac{2}{3}d\lambda[\sigma_x - \frac{1}{2}(\sigma_y + \sigma_z)]$$

$$d\varepsilon_y = \frac{2}{3}d\lambda[\sigma_y - \frac{1}{2}(\sigma_x + \sigma_z)],$$

$$d\varepsilon_z = \frac{2}{3}d\lambda[\sigma_z - \frac{1}{2}(\sigma_y + \sigma_x)],$$

$$d\varepsilon_{xy} = d\lambda\tau_{xy}, \quad d\varepsilon_{yz} = d\lambda\tau_{yz}, \quad d\varepsilon_{xz} = d\lambda\tau_{xz}$$

Tasomuodonmuutostilassa:

$$d\varepsilon_z = 0$$

$$\sigma_z = \frac{1}{2}(\sigma_y + \sigma_x)$$

$$p = \frac{1}{2}(\sigma_y + \sigma_x) = \sigma_z$$

$$S_x = \frac{1}{2}(\sigma_x - \sigma_y),\ S_y = \frac{1}{2}(\sigma_y - \sigma_x),\ S_{xy} = \tau_{xy},$$

$$S_{yz} = 0,\ S_{xz} = 0,\ S_z = 0.$$

Tasojännitystilassa:

$$d\varepsilon_z \neq 0, \text{ mutta } \sigma_z = 0$$

$$\frac{1}{2}S_{ij}S_{ij} = k^2$$

$$\frac{1}{2}(S_xS_x + S_yS_y + S_{xy}S_{xy} + S_{yx}S_{yx}) = k^2$$

$$\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}^2 = k^2$$

$$k = \sigma_m/\sqrt{3}$$

$$\det\left(\begin{bmatrix} \sigma_x - \sigma & \tau_{xy} \\ \tau_{xy} & \sigma_y - \sigma \end{bmatrix}\right) = 0$$

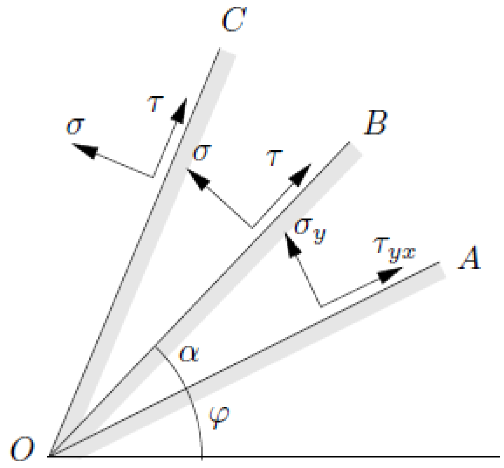
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_3 = \sigma_z = \frac{1}{2}(\sigma_x + \sigma_y) = p$$

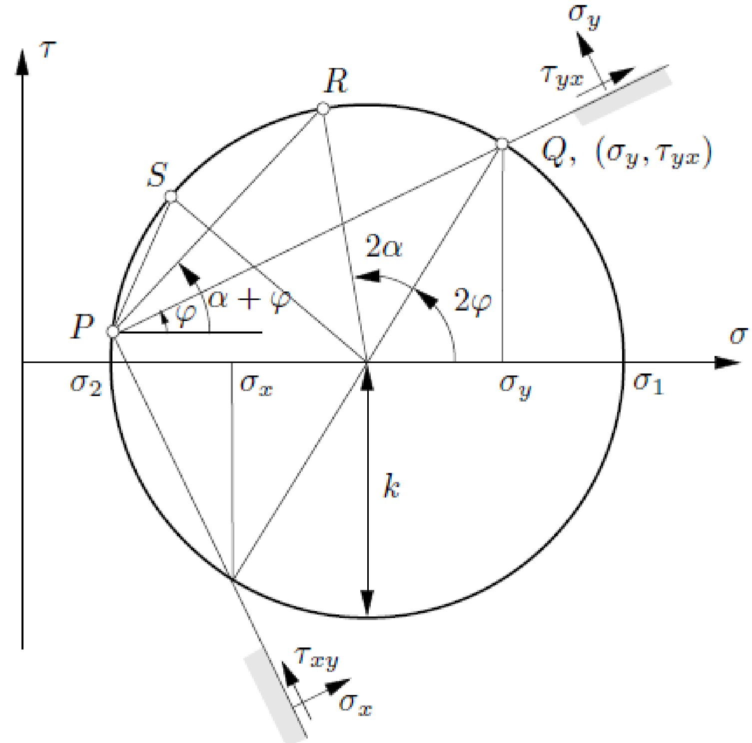
$$\frac{1}{2}|\sigma_1 - \sigma_2| = k$$

$$\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}^2 = k^2$$

$$k = \sigma_m/2$$



$PQ||OA$
$PR||OB$
$PS||OC$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$r_M = \frac{1}{2}(\sigma_1 - \sigma_2)$$

$$O_M := \left\{\frac{1}{2}(\sigma_1 + \sigma_2), 0\right\}$$

$$\frac{1}{2}(\sigma_1 - \sigma_2) \leq k$$

$$r_M = k$$

$$\sigma_x = p - k\cos 2\varphi$$

$$\sigma_y = p + k\cos 2\varphi$$

$$\tau_{xy} = k\sin 2\varphi$$

$$p = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(\sigma_1 + \sigma_2)$$

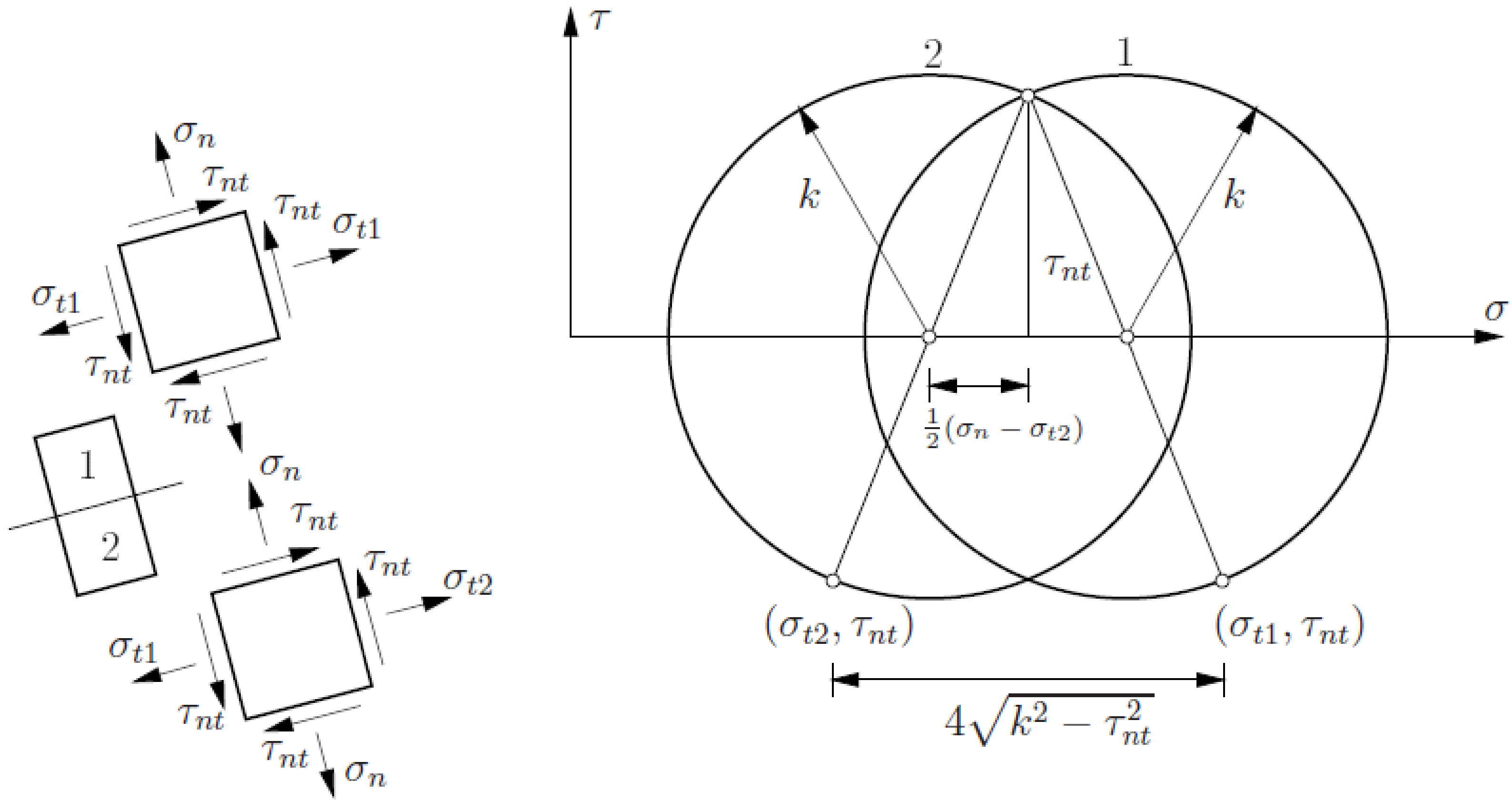
$$k = \frac{1}{2}(\sigma_1 - \sigma_2) = \tau_{\max} = \tau_m$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

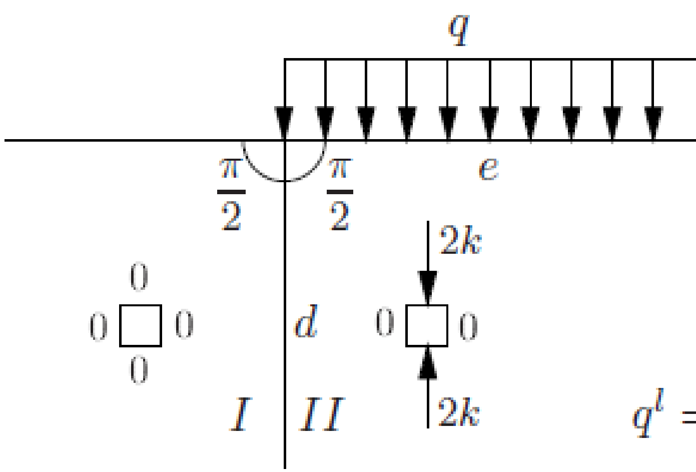
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$$

$$\left(\frac{\sigma_{t1} - \sigma_n}{2}\right)^2 + \tau_{nt}^2 = \left(\frac{\sigma_n - \sigma_{t2}}{2}\right)^2 + \tau_{nt}^2 = k^2$$

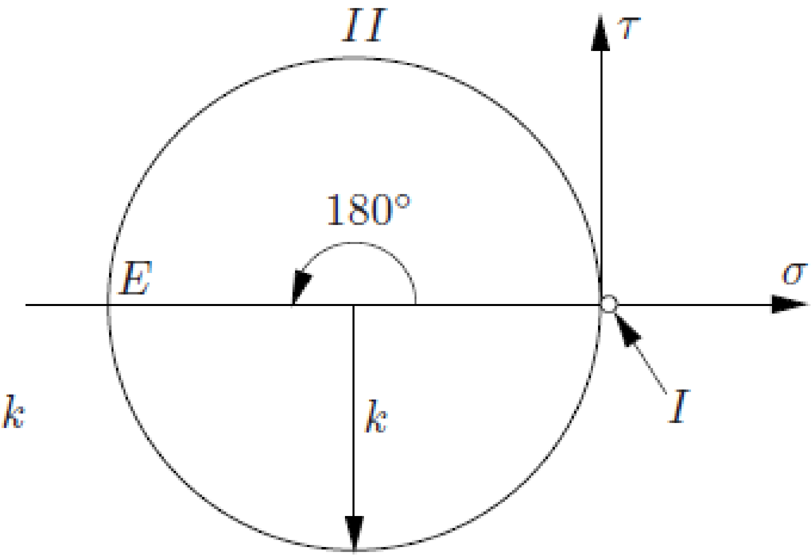
$$\sigma_{t1} - \sigma_{t2} = 4\sqrt{k^2 - \tau_{nt}^2}$$

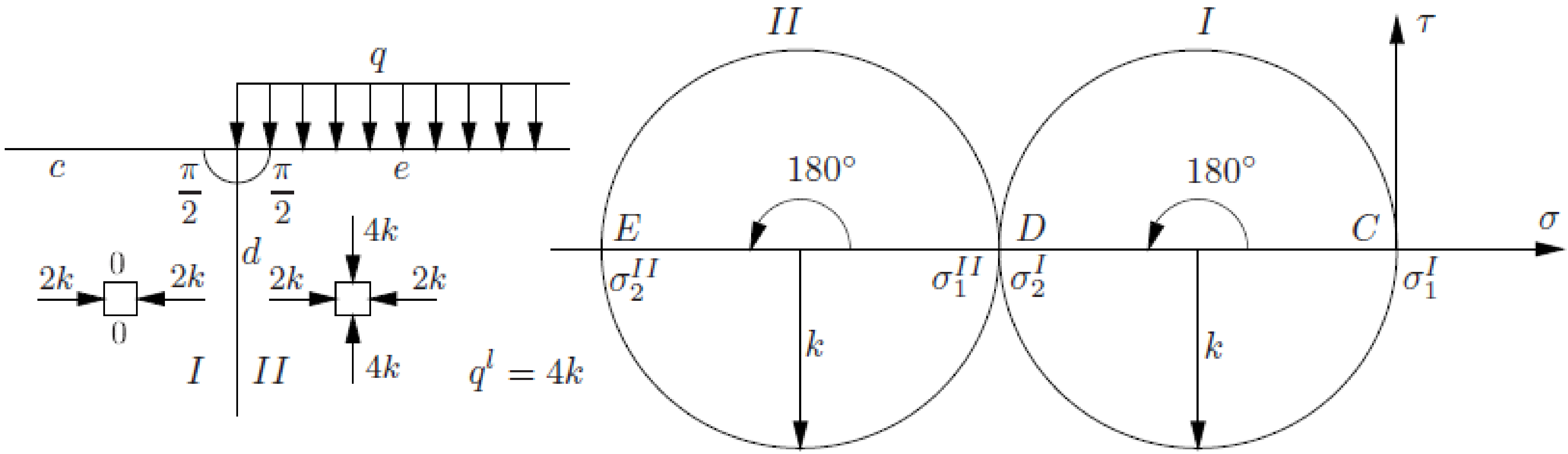


σ_n
τ_{nt}
τ_{nt}
σ_{t1}
σ_{t1}
τ_{nt}
τ_{nt}
1
2
σ_n
τ_{nt}
τ_{nt}
σ_{t2}
σ_{t1}
τ_{nt}
τ_{nt}
σ_n
τ
2
1
k
k
τ_{nt}
σ
$\frac{1}{2}(\sigma_n - \sigma_{t2})$
(σ_{t2}, τ_{nt})
(σ_{t1}, τ_{nt})
$4\sqrt{k^2 - \tau_{nt}^2}$

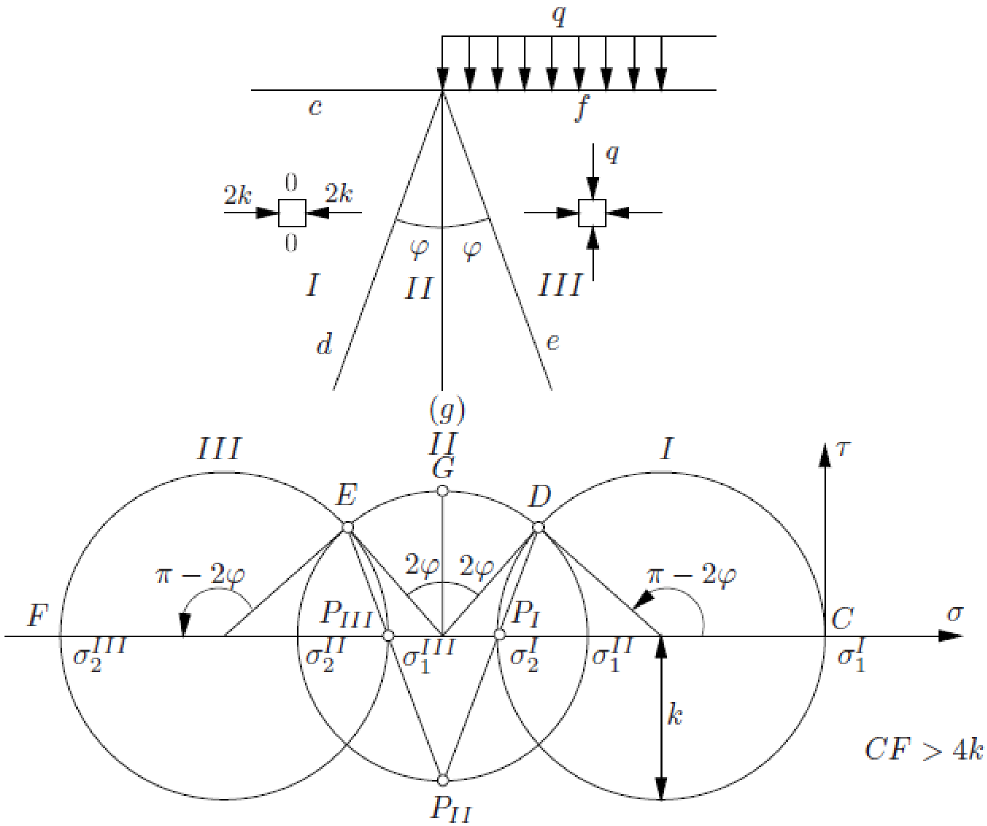


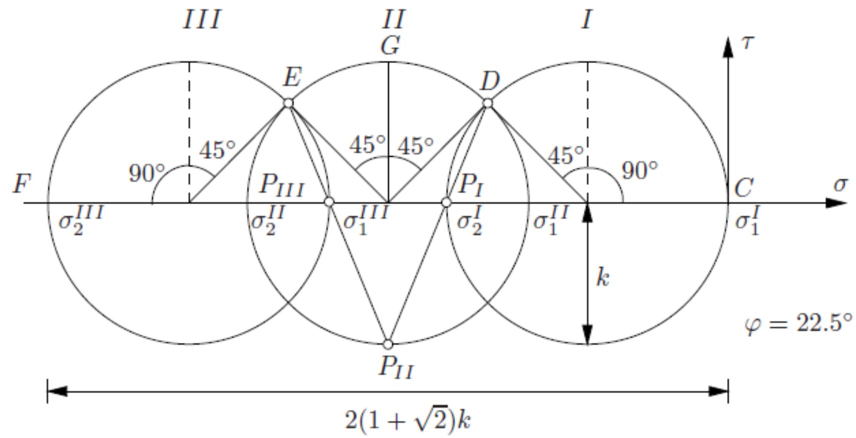
$q^l = 2k$

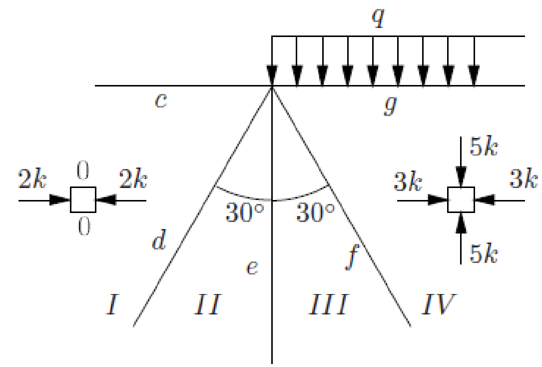



q
c
π/2
π/2
e
d
4k
2k
0
2k
2k
2k
0
4k
I
II
$q^l = 4k$
II
I
τ
180°
180°
E
D
C
σ
σ_2^{II}
σ_1^{II}
σ_2^{I}
σ_1^{I}
k
k

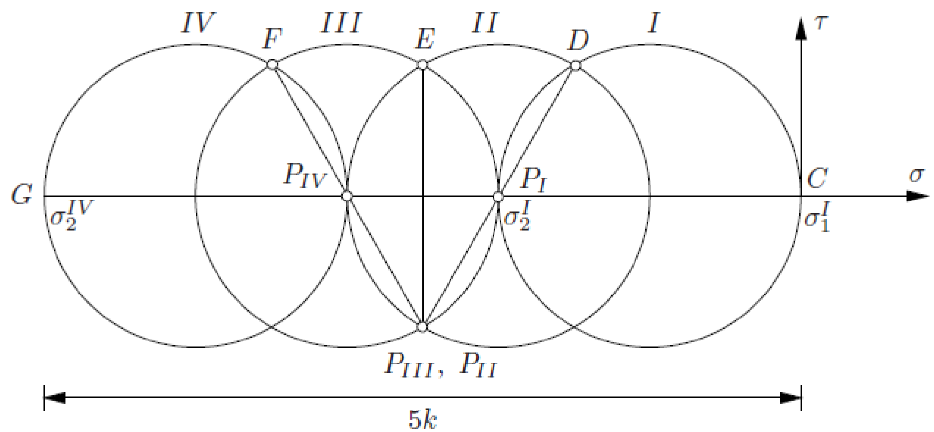
$$q^l = (2 + 2\sqrt{2})k \approx 4.83k.$$

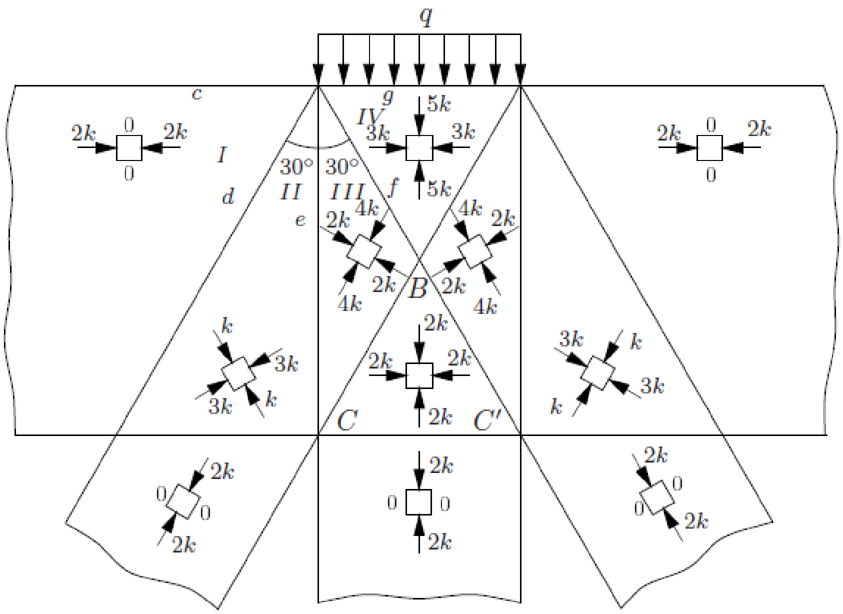


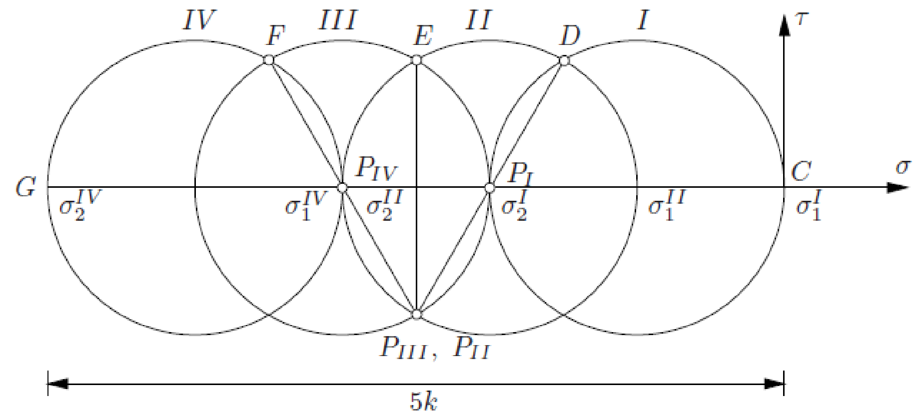


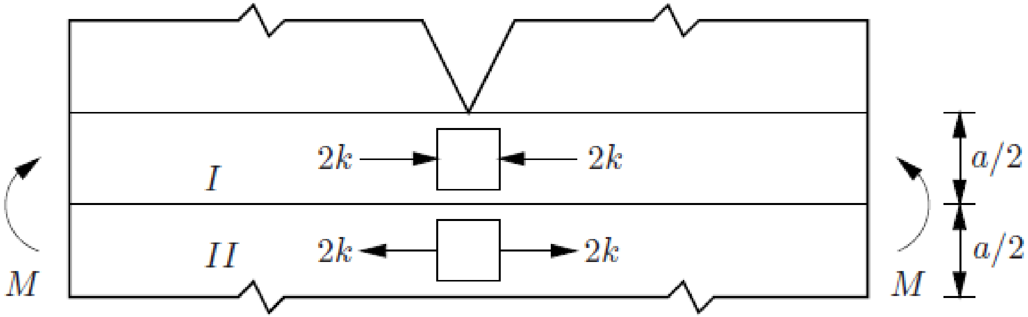

q
c
g
2k
0
2k
0
30°
30°
5k
3k
3k
5k
d
e
f
I
II
III
IV

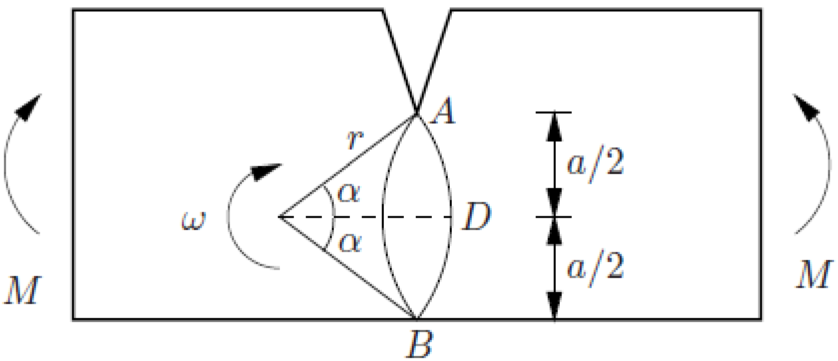
$$q^l = 5k$$

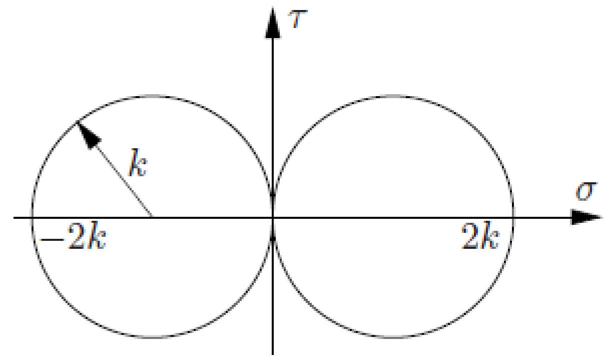

IV
F
III
E
II
D
I
τ
G
P_{IV}
P_I
C
σ
σ_2^{IV}
σ_2^I
σ_1^I
P_{III}, P_{II}
5k

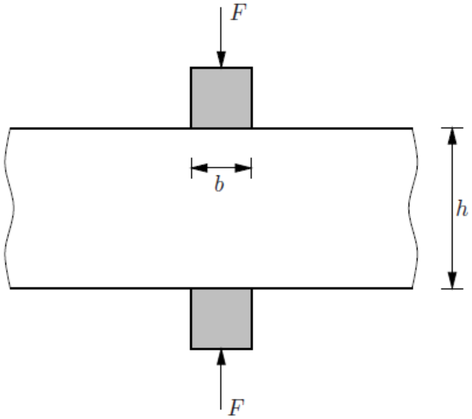

q
c
g
IV
5k
3k
3k
2k
2k
0
0
I
d
30°
30°
II
III
e
f
4k
2k
B
k
3k
C
C′


IV
F
III
E
II
D
I
τ
G
P_{IV}
P_I
C
σ
σ_2^{IV}
σ_1^{IV}
σ_2^{II}
σ_2^{I}
σ_1^{II}
σ_1^{I}
P_{III}, P_{II}
5k

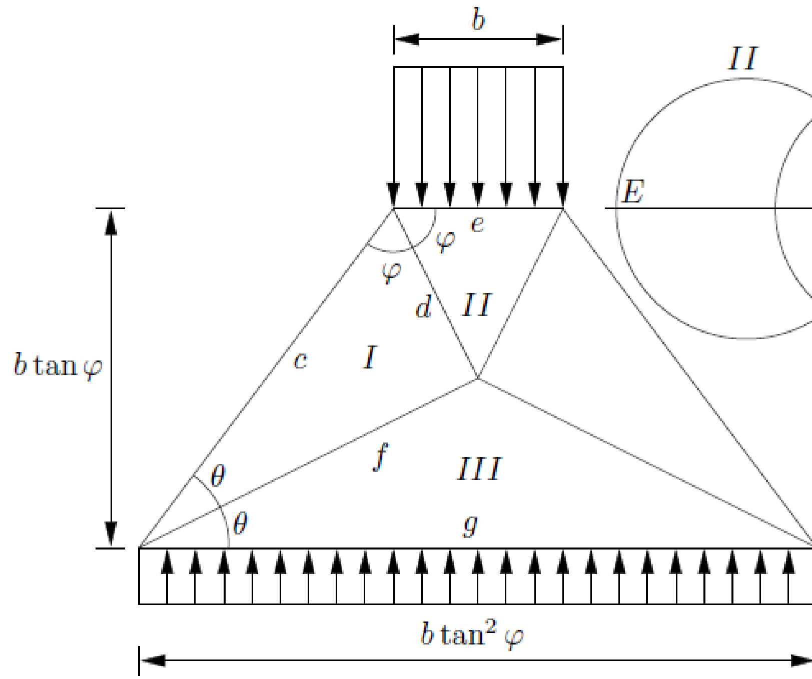

2k
2k
I
II
2k
2k
M
M
a/2
a/2

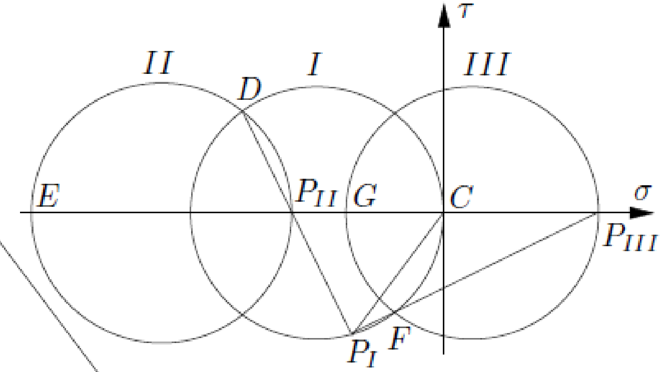

A
r
α
α
ω
D
B
M
M
a/2
a/2


τ
k
σ
−2k
2k



$$F^l = \frac{4kb}{1 + \left(\frac{2b}{h}\right)^2}$$

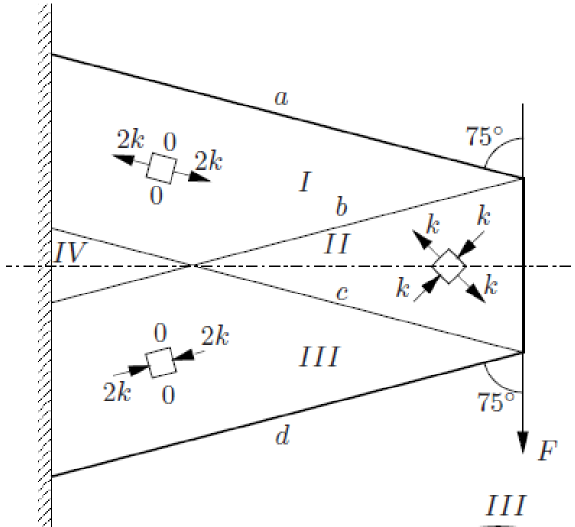


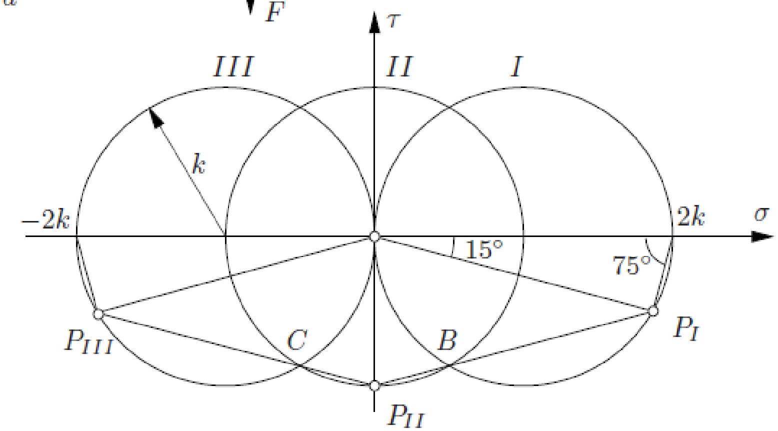


$45° \leq \varphi \leq 90°$

$\tan\varphi = \dfrac{h}{2b}$

$\sin\varphi \geq \cos\varphi$


a
b
c
d
I
II
III
IV
75°
75°
2k
2k
0
0
k
k
k
k
0
2k
2k
0
F


τ
σ
III
II
I
k
−2k
2k
15°
75°
P_I
P_{II}
P_{III}
C
B