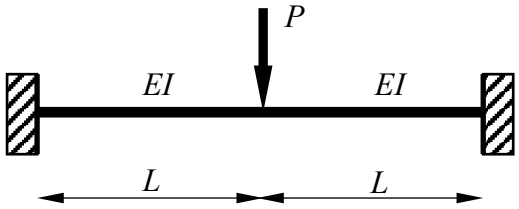
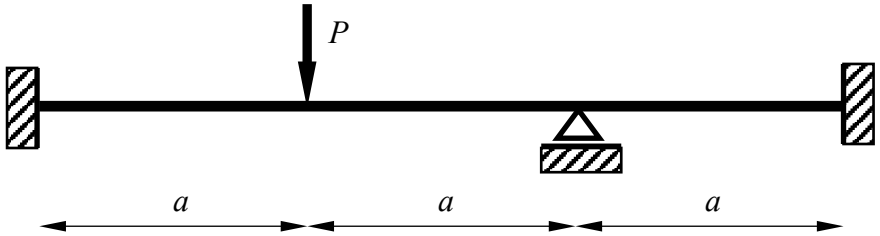
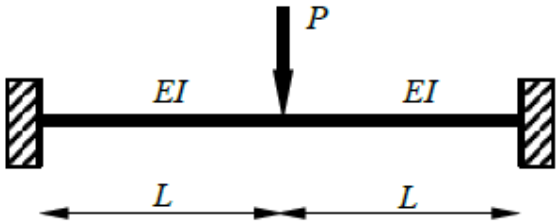


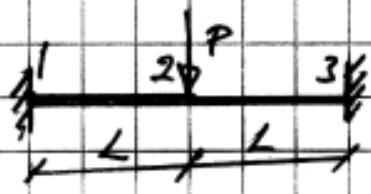
1. Ratkaise oheisen tehtävän siirtymät plastisessa rajatilassa. Laske erityisesti voiman *P* pystysiirtymä. Päättele missä järjestyksessä plastiset nivelet syntyvät.

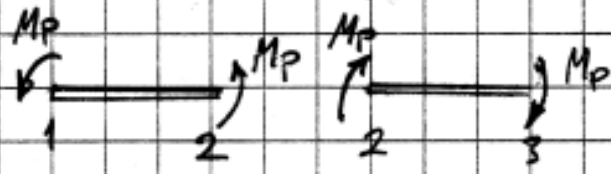


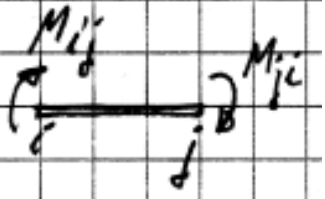
2. Ratkaise yllä olevan tehtävän siirtymät plastisessa rajatilassa. *EI* on vakio koko palkin alueella. Missä järjestyksessä plastiset nivelet syntyvät.



1. Ratkaise oheisen tehtävän siirtymät plastisessa rajatilassa. Laske erityisesti voiman P pystysiirtymä. Päättele missä järjestyksessä plastiset nivelet syntyvät.







$4M_p\theta = PL\theta \quad \forall\theta \qquad P = 4\frac{M_p}{L}$

$M_{12} = -M_p \qquad M_{21} = -M_p$

$M_{23} = M_p \qquad M_{32} = M_p$

$$\varphi_{12} = \frac{L}{3EI}(-M_p) - \frac{L}{6EI}(-M_p) + \frac{v}{L} \Rightarrow \varphi_{12} = -\frac{M_pL}{6EI} + \frac{v}{L}$$

$$\varphi_{21} = \frac{L}{3EI}(-M_p) - \frac{L}{6EI}(-M_p) + \frac{v}{L} \Rightarrow \varphi_{21} = -\frac{M_pL}{6EI} + \frac{v}{L}$$

$$\varphi_{23} = \frac{L}{3EI}M_p - \frac{L}{6EI}M_p - \frac{v}{L} \Rightarrow \varphi_{23} = \frac{M_pL}{6EI} - \frac{v}{L}$$

$$\varphi_{32} = \frac{L}{3EI}M_p - \frac{L}{6EI}M_p - \frac{v}{L} \Rightarrow \varphi_{32} = \frac{M_pL}{6EI} - \frac{v}{L}$$

a) Vilmeinen nivel pisteeseen 1

$$\varphi_{12} = 0 \Rightarrow \frac{v}{L} = \frac{M_pL}{6EI}$$

$$\Rightarrow v = \frac{M_pL^2}{6EI}$$

$$\theta_2 = \varphi_{21} - \varphi_{23} = -\frac{M_pL}{6EI} + \frac{v}{L} - \frac{M_pL}{6EI} + \frac{v}{L}$$

$$= -\frac{2M_pL}{6EI} + \frac{2M_pL}{6EI} = 0$$

b/ Viimeinen nivel pisteessä 2

$$\varphi_{21} = \varphi_{23}$$

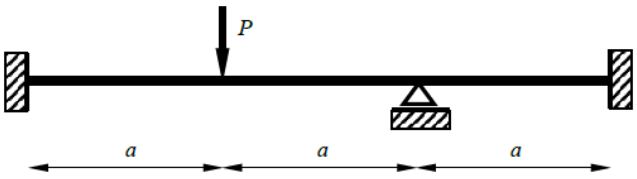
$$-\frac{M_p L}{6EI} + \frac{v}{L} = \frac{M_p L}{6EI} - \frac{v}{L} \Rightarrow v = \frac{1}{6}\frac{M_p L^2}{EI}$$

$$\theta_1 = -\varphi_{12} = \frac{M_p L}{6EI} - \frac{v}{L} = \frac{M_p L}{6EI} - \frac{M_p L}{6EI} = 0$$

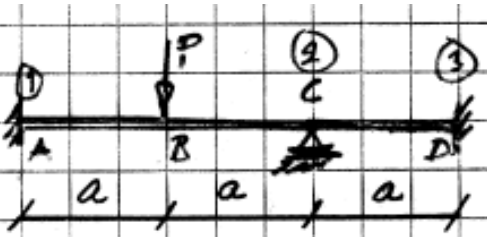
$$\theta_3 = \varphi_{32} = \frac{M_p L}{6EI} - \frac{v}{L} = \frac{M_p L}{6EI} - \frac{M_p L}{6EI} = 0$$

a/ ja b/ antaa saman tapauksen.

$$v = \frac{M_p L^2}{6EI} = \frac{1}{4}\frac{PL \cdot L^2}{6EI} = \frac{PL^3}{24EI}$$

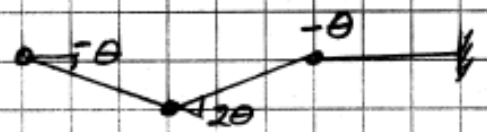


2. Ratkaise yllä olevan tehtävän siirtymät plastisessa rajatilassa. *EI* on vakio koko palkin alueella. Missä järjestyksessä plastiset nivelet syntyvät.

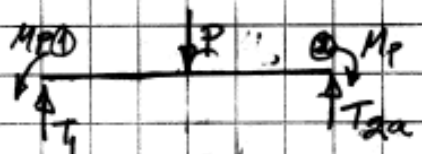


M_p vakio
EI vakio

$W_u = P a \theta$



$W_s = M_p\theta + M_p 2\theta + M_p\theta = 4M_p\theta$



$\Rightarrow P_p = 4\frac{M_p}{a}$

$\circlearrowleft 2: \; -T_1 2a + M_p + Pa - M_p = 0$

$T_1 = \frac{P}{2} = 2\frac{M_p}{a}$

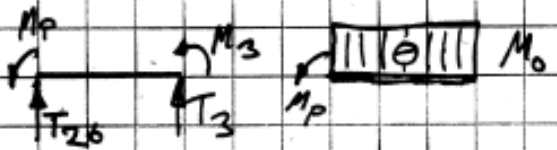
$\varphi_{ij} = \frac{L}{3EI} M_{ij} - \frac{L}{6EI} M_{ji} + \psi_{ij}$

$\varphi_{ji} = -\frac{L}{6EI} M_{ij} + \frac{L}{3EI} M_{ji} + \psi_{ij}$

$\psi_{ij} = \frac{v}{L}$

$\Sigma F_y: \; 2\frac{M_p}{a} - P + T_{2a} = 0$

$T_{2a} = P - 2\frac{M_p}{a} = 4\frac{M_p}{a} - 2\frac{M_p}{a} = 2\frac{M_p}{a}$



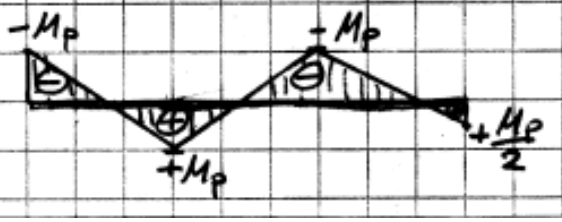
olkoon $T_{2b} = 1$

$\delta_{10} = \frac{1}{2} a \cdot (-M_p a) = -\frac{M_p a^2}{2}$

$\delta_{11} = \frac{1}{3} a^3 = \frac{a^3}{3}$

$\delta_{10} + \delta_{11} T_{2b} = 0 \qquad T_{2b} = \frac{3}{2}\frac{M_p}{a}$

$M_3 = T_{2b} a - M_p = \frac{1}{2} M_p$



tai $\varphi_{32} = 0$

$\frac{a}{3EI} M_{32} - \frac{a}{6EI}(-M_p) = 0$

$M_{32} = -\frac{1}{2} M_p$

Oletetaan, että viimeinen nivel syntyy pisteessä 2 "P" on piste P, jossa on voima

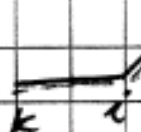
$\Rightarrow \varphi_{2P} = \varphi_{23}$ $\quad M_{2P} = M_P \quad M_{P2} = M_P \quad \psi_{2P} = \frac{\delta}{a}$

$M_{23} = -M_P \quad M_{32} = -\frac{1}{2}M_P$

$$\varphi_{2P} = \frac{a}{3EI} M_{2P} - \frac{a}{6EI} M_{P2} + \psi_{2P} = \frac{a}{6EI}(2M_P - M_P) - \frac{\delta}{a}$$

$$\varphi_{23} = \frac{a}{3EI} M_{23} - \frac{a}{6EI} M_{32} = \frac{a}{6EI}\left(-2M_P + \frac{1}{2}M_P\right)$$

$$\Rightarrow \frac{\delta}{a} = \frac{a}{6EI}\left(M_P + \frac{3}{2}M_P\right) \quad \delta = \frac{5}{12}\frac{M_P a^2}{EI}$$

$\theta_i = \varphi_{ik} - \varphi_{il}$ 

Solmu 1 $\quad \theta_1 = -\varphi_{1P} = -\left[\frac{a}{3EI}(-M_P) - \frac{a}{6EI}(-M_P) + \frac{\delta}{a}\right]$

$$= \frac{a}{6EI}M_P + \frac{5}{12}\frac{a}{EI}M_P = -\frac{3}{12}\frac{M_P a}{EI} < 0$$

Solmu P $\quad \theta_P = \varphi_{P1} - \varphi_{P2} = \frac{a}{3EI}(-M_P) - \frac{a}{6EI}(-M_P) + \frac{\delta}{a} +$

$$-\left[\frac{a}{3EI}M_P - \frac{a}{6EI}M_P - \frac{\delta}{a}\right.$$

$$= \frac{a}{6EI}\left[-4M_P + 2M_P\right] + 2\frac{\delta}{a}$$

$$= \frac{3}{6}\frac{M_P a}{EI} > 0 \quad \checkmark$$

Solmu 2 $\quad \theta_2 = \varphi_{2P} - \varphi_{23} = 0$

$$\varphi_{2P} = \frac{a}{3EI}M_P - \frac{a}{6EI}M_P - \frac{\delta}{a} = -\frac{1}{4}\frac{M_P a}{EI}$$

$$\varphi_{23} = \frac{a}{3EI}(-M_P) - \frac{a}{6EI}\left(-\frac{1}{2}M_P\right) = -\frac{1}{4}\frac{M_P a}{EI} \quad \underline{\underline{\text{ok}}}$$