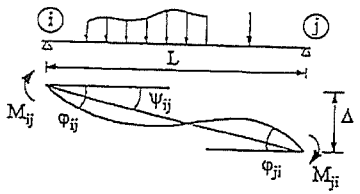
PLASTISUUSTEORIA , Kaavakokoelma

15.10.1998

$$\varphi_{ij} = \alpha_{ij} M_{ij} - \beta_{ij} M_{ji} + \psi_{ij} + \alpha^{o}_{ij}$$

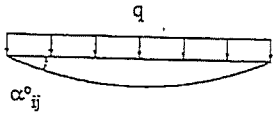
$$\varphi_{ji} = -\beta_{ji} M_{ij} + \alpha_{ji} M_{ji} + \psi_{ji} + \alpha^{o}_{ji}$$



Kun taivutusjäykkyys EI = vakio, niin

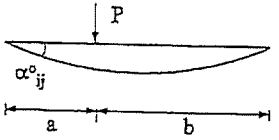
$$\alpha_{ij} = \frac{L}{3EI} = \alpha_{ji}\,,\ \beta_{ij} = \frac{L}{6EI} = \beta_{ji}\,,\ \psi_{ij} = \psi_{ji} = \frac{\Delta}{L}$$

Tasainen kuorma q:



$$\alpha^{0}_{ij} = \frac{qL^3}{24EI} = \alpha^{0}_{ji}$$

Pistekuorma kohdassa x = a :

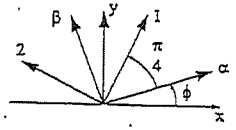


$$\alpha^{0}_{ij} = \frac{Pab}{6EI}\left(1+\frac{b}{L}\right)$$

$$\alpha^{0}_{ji} = -\frac{Pab}{6EI}\left(1+\frac{a}{L}\right)$$

Tasomuodonmuutostila:

Hencky Geiringer

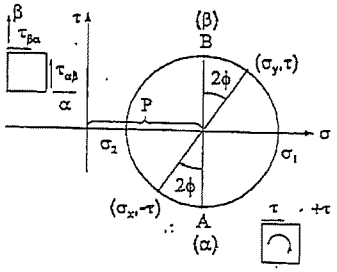


$p - 2k\phi = c_\alpha \qquad dv_\alpha - v_\beta d\phi = 0 \quad \alpha$:lla

$p + 2k\phi = c_\beta \qquad dv_\beta + v_\alpha d\phi = 0 \quad \beta$:lla

$$p = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(\sigma_1 + \sigma_2)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$



$\sigma_1 = p + k\,,\ \sigma_2 = p - k\,,\ \sigma_3 = p$

$\sigma_x = p - k \sin 2\phi$

$\sigma_y = p + k \sin 2\phi$

$\tau_{xy} = k \cos 2\phi$

von Mises & Tresca

$$\sigma_1 - \sigma_2 \le 2k \qquad \frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau^2_{xy} \le k^2$$

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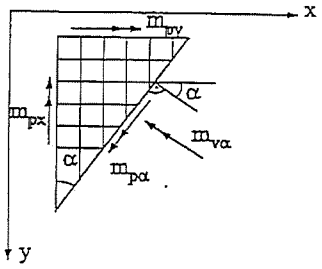
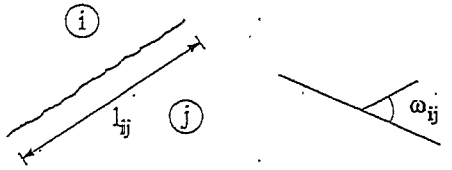
Teräsbetonilaatta



$$m_{p\alpha} = m_{px}\cos^2\alpha + m_{py}\sin^2\alpha$$

Dissipaatio myötöviivalla

$$D_{ij} = (m_p)_{ij}\, l_{ij}\, \omega_{ij}$$



tai

$$D = |\bar{m}\cdot\bar{w}|\,\text{sign}$$

$\text{sign} = 1$, jos $\bar{m}\cdot\bar{w} < 0$

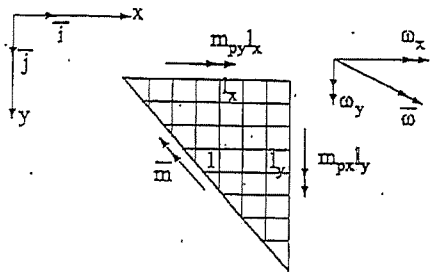
$\text{sign} = -1$, jos $\bar{m}\cdot\bar{w} > 0$

eli

$$D = (m_{px} l_y w_y + m_{py} l_x w_x)\,\text{sign}$$

$$\bar{w} = w_x\,\bar{i} + w_y\,\bar{j}$$

$$\bar{m} = m_{px} l_y\,\bar{i} + m_{py} l_x\,\bar{j}$$



$$M_m = W\sigma_m \quad W = \frac{I}{e_r}$$

$$M_p = W_p\sigma_m \quad W_p = A_1z_1 + A_2z_2$$

$$\sum |M_{pi}\theta_i| = \sum F_k\Delta_k$$

$$\theta_j = \varphi_{ji} - \varphi_{jk}$$

$$G = k\sum_i M_{pi}L_i$$

$$M_{zp} = max\left(2\int_A \emptyset dA\right), \quad |\nabla\emptyset| \le \tau_m$$

$$M_{zp} = 2At_{min}\tau_m$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\frac{\partial^2 m_x}{\partial x^2} + 2\frac{\partial^2 m_{xy}}{\partial x\partial y} + \frac{\partial^2 m_y}{\partial y^2} = -p$$

$$\sum_{ij} \tau_m l_{ij} v_{ij} + 2k|\dot{\varepsilon}|A + k\dot{\gamma}A = Fv_F$$

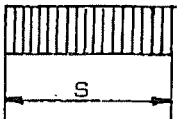
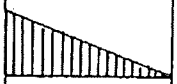
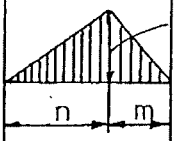
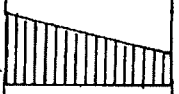
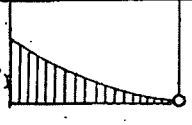
$$\{\hat{u}\} = [a]\{X\} + \{u_0\}$$

$$a_{ij} = \int_s \frac{\overline{M}_{ti}\,\overline{M}_{tj}}{EI}\,ds$$

$$u_{i0} = \int_s \frac{M_{t0}\,\overline{M}_{ti}}{EI}\,ds$$

Taulukko 7.1

$\int_0 M\bar{M}ds$

$\bar{M}$-pinta / M-pinta	(a)	(b)	(c)	(d)
(1) 	$s \cdot y \cdot \bar{y}$	$\frac{1}{2} s \cdot y \cdot \bar{y}_2$	$\frac{1}{2} \cdot s \cdot y \cdot \bar{y}_3$	$\frac{1}{2} sy(\bar{y}_1 + \bar{y}_2)$
(2) 	$\frac{1}{2} \cdot s \cdot y_2 \cdot \bar{y}$	$\frac{1}{3} s \cdot y_2 \cdot \bar{y}_2$	$\frac{1}{6}(s+n) y_2 \bar{y}_3$	$\frac{1}{6} sy_2 \cdot (\bar{y}_1 + 2\bar{y}_2)$
(3) 	$\frac{1}{2} \cdot s \cdot y_1 \cdot \bar{y}$	$\frac{1}{6} \cdot s \cdot y_1 \cdot \bar{y}_2$	$\frac{1}{6}(s+m) y_1 \bar{y}_3$	$\frac{1}{6} \cdot s \cdot y_1 \cdot (2\bar{y}_1 + \bar{y}_2)$
(4) 	$\frac{1}{2} \cdot s \cdot y_3 \cdot \bar{y}$	$\frac{1}{6}(s+n) y_3 \bar{y}_2$	$\frac{1}{3} \cdot s \cdot y_3 \cdot \bar{y}_3$	$\frac{1}{6} y_3 \cdot [(s+m) \cdot \bar{y}_1 + (s+n) \cdot \bar{y}_2]$
(5) 	$\frac{s}{2}(y_1 + y_2)\bar{y}$	$\frac{s}{6}(y_1 + 2y_2)\bar{y}_2$	$\frac{\bar{y}_3}{6} \cdot [(s+m) y_1 + (s+n) y_2]$	$\frac{s}{6}[y_1(2\bar{y}_1 + \bar{y}_2) + y_2(\bar{y}_1 + 2\bar{y}_2)]$
(6) 	$\frac{2}{3} \cdot s \cdot y_3 \cdot \bar{y}$	$\frac{1}{3} s y_3 \bar{y}_2$	$\frac{y_3 \bar{y}_3}{3s}(s^2 + nm)$	$\frac{sy_3}{3}(\bar{y}_1 + \bar{y}_2)$
(7) 	$\frac{2}{3} \cdot s \cdot y_2 \cdot \bar{y}$	$\frac{5}{12} \cdot s \cdot y_2 \cdot \bar{y}_2$	$\frac{y_2 \bar{y}_3}{12 \cdot s} \cdot (5s^2 - ms - m^2)$	$\frac{sy_2}{12}(3\bar{y}_1 + 5\bar{y}_2)$
(8) 	$\frac{2}{3} \cdot s \cdot y_1 \cdot \bar{y}$	$\frac{1}{4} \cdot s \cdot y_1 \cdot \bar{y}_2$	$\frac{y_1 \bar{y}_3}{12 \cdot s} \cdot (5s^2 - ns - n^2)$	$\frac{sy_1}{12} \cdot (5\bar{y}_1 + 3\bar{y}_2)$
(9) 	$\frac{1}{3} \cdot s \cdot y_2 \cdot \bar{y}$	$\frac{1}{4} \cdot s \cdot y_2 \cdot \bar{y}_2$	$\frac{y_2 \bar{y}_3}{12s} \cdot (s^2 + ns + n^2)$	$\frac{sy_2}{12}(\bar{y}_1 + 3\bar{y}_2)$
(10) 	$\frac{1}{3} \cdot s \cdot y_1 \cdot \bar{y}$	$\frac{1}{12} \cdot s\ y_1 \cdot \bar{y}_2$	$\frac{y_1 \bar{y}_3}{12 \cdot s} \cdot (s^2 + ms + m^2)$	$\frac{sy_1}{12} \cdot (3\bar{y}_1 + \bar{y}_2)$
(11) $\int \bar{M}^2 ds$	$s \cdot \bar{y}^2$	$\frac{1}{3} \cdot s \cdot \bar{y}_2^2$	$\frac{1}{3} \cdot s \cdot \bar{y}_3^2$	$\frac{1}{3} s(\bar{y}_1^2 + \bar{y}_1 \bar{y}_2 + \bar{y}_2^2)$

Trigonometristen funktioiden tarkkoja arvoja

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asteet	radiaanit	sin	cos	tan	cot
0	0	0	1	0	—
15	$\frac{\pi}{12}$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$2-\sqrt{3}$	$2+\sqrt{3}$
18	$\frac{\pi}{10}$	$\frac{1}{4}(\sqrt{5}-1)$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{1}{5}\sqrt{25-10\sqrt{5}}$	$\sqrt{5+2\sqrt{5}}$
22,5	$\frac{\pi}{8}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\sqrt{2}-1$	$\sqrt{2}+1$
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
36	$\frac{\pi}{5}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{1}{4}(\sqrt{5}+1)$	$\sqrt{5-2\sqrt{5}}$	$\frac{1}{5}\sqrt{25+10\sqrt{5}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1
54	$\frac{3\pi}{10}$	$\frac{1}{4}(\sqrt{5}+1)$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{1}{5}\sqrt{25+10\sqrt{5}}$	$\sqrt{5-2\sqrt{5}}$
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
67,5	$\frac{3\pi}{8}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\sqrt{2}+1$	$\sqrt{2}-1$
72	$\frac{2\pi}{5}$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{1}{4}(\sqrt{5}-1)$	$\sqrt{5+2\sqrt{5}}$	$\frac{1}{5}\sqrt{25-10\sqrt{5}}$
75	$\frac{5\pi}{12}$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$2+\sqrt{3}$	$2-\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	—	0
105	$\frac{7\pi}{12}$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$-\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$-(2+\sqrt{3})$	$\sqrt{3}-2$
108	$\frac{3\pi}{5}$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$-\frac{1}{4}(\sqrt{5}-1)$	$-\sqrt{5+2\sqrt{5}}$	$-\frac{1}{5}\sqrt{25-10\sqrt{5}}$
112,5	$\frac{5\pi}{8}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$-\frac{1}{2}\sqrt{2-\sqrt{2}}$	$-\sqrt{3+2\sqrt{2}}$	$-\sqrt{3-2\sqrt{2}}$
120	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$
126	$\frac{7\pi}{10}$	$\frac{1}{4}(\sqrt{5}+1)$	$-\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$-\frac{1}{5}\sqrt{25+10\sqrt{5}}$	$-\sqrt{5-2\sqrt{5}}$
135	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1	-1
144	$\frac{4\pi}{5}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$-\frac{1}{4}(\sqrt{5}+1)$	$-\sqrt{5-2\sqrt{5}}$	$-\frac{1}{5}\sqrt{25+10\sqrt{5}}$
150	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$
157,5	$\frac{7\pi}{8}$	$\frac{1}{2}\left(\sqrt{2-\sqrt{2}}\right)$	$-\frac{1}{2}\sqrt{2+\sqrt{2}}$	$1-\sqrt{2}$	$-(\sqrt{2}+1)$
162	$\frac{9\pi}{10}$	$\frac{1}{4}(\sqrt{5}-1)$	$-\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$-\frac{1}{5}\sqrt{25-10\sqrt{5}}$	$-\sqrt{5+2\sqrt{5}}$
165	$\frac{11\pi}{12}$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$-\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$\sqrt{3}-2$	$-(2+\sqrt{3})$
180	π	0	-1	0	—
195	$\frac{13\pi}{12}$	$-\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$-\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$2-\sqrt{3}$	$2+\sqrt{3}$
210	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
225	$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1	1
240	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
255	$\frac{17\pi}{12}$	$-\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$-\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$2+\sqrt{3}$	$2-\sqrt{3}$
270	$\frac{3\pi}{2}$	-1	0	—	0
285	$\frac{19\pi}{12}$	$-\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$-(2+\sqrt{3})$	$\sqrt{3}-2$
300	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$
315	$\frac{7\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1	-1
330	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$
345	$\frac{23\pi}{12}$	$-\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$\sqrt{3}-2$	$-(2+\sqrt{3})$
360	2π	0	1	0	—

Palautuskaavat

15. $\sin x = -\sin(-x) = \cos\left(\frac{\pi}{2} - x\right) = \sin(\pi - x) = \sin(x + n2\pi)$

16. $\cos x = \cos(-x) = \sin\left(\frac{\pi}{2} - x\right) = -\cos(\pi - x) = \cos(x + n2\pi)$

17. $\tan x = -\tan(-x) = \cot\left(\frac{\pi}{2} - x\right) = -\tan(\pi - x) = \tan(x + n\pi)$

18. $\cot x = -\cot(-x) = \tan\left(\frac{\pi}{2} - x\right) = -\cot(\pi - x) = \cot(x + n\pi)$

Kaksinkertaiset kulmat

19. $\sin 2x = 2 \sin x \cos x$

20. $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

21. $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{2}{\cot x - \tan x}$

22. $\cot 2x = \frac{\cot^2 x - 1}{2 \cot x} = \frac{\cot x - \tan x}{2}$

Kolminkertaiset kulmat

23. $\sin 3x = 3 \sin x - 4 \sin^3 x = \sin x (4 \cos^2 x - 1)$

24. $\cos 3x = 4 \cos^3 x - 3 \cos x = \cos x (1 - 4 \sin^2 x)$

25. $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

26. $\cot 3x = \frac{\cot^3 x - 3 \cot x}{3 \cot^2 x - 1}$

Puolikkaat kulmat

Etumerkkivaihtoehdoista valitaan edellisen sivun merkkikaavion mukainen kyseiseen neljännekseen kuuluva etumerkki.

27. $\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$

28. $\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$

29. $\tan \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$

30. $\cot \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{1 - \cos x}} = \frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$

Trigonometristen funktioiden potensseja

31. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

32. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

33. $\sin^3 x = \frac{1}{4}(3 \sin x - \sin 3x)$

34. $\cos^3 x = \frac{1}{4}(\cos 3x + 3 \cos x)$

35. $\sin^4 x = \frac{1}{8}(\cos 4x - 4 \cos 2x + 3)$

36. $\cos^4 x = \frac{1}{8}(\cos 4x + 4 \cos 2x + 3)$

Summakaavoja

Kaksoismerkeistä ylemmät vastaavat toisiaan, samoin alemmat.

37. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

38. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

39. $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

40. $\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$

41. $\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$

42. $\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$

43. $\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$

44. $\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$

45. $\tan x \pm \tan y = \frac{\sin(x \pm y)}{\cos x \cos y}$

46. $\cot x \pm \cot y = \frac{\sin(y \pm x)}{\sin x \sin y}$

47. $\cos x \pm \sin x = \sqrt{2} \sin\left(\frac{\pi}{4} \pm x\right) = \sqrt{2} \cos\left(\frac{\pi}{4} \mp x\right)$

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