Introduction to materials modelling

Lecture 11 - Viscoelasticity, creep

Reijo Kouhia

Tampere University, Structural Mechanics

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Viscoelasticity

Some materials show pronounced influence of the rate of loading. Metals at elevated temperatures, concrete, plastics.

Simple models can be build by using elastic spring and viscous dashpot models:

\[
\begin{align*}
\text{elastic spring} \quad \sigma &= E\varepsilon, \\
\text{viscous dashpot} \quad \sigma &= \eta \frac{d\varepsilon}{dt} = \eta \dot{\varepsilon}
\end{align*}
\]

Study of flow of matter is called **rheology**.
Creep and relaxation

- **Creep:** increase of strain when the specimen is loaded by a constant stress.
- **Relaxation:** decrease of stress when the strain is kept constant.
Basic Maxwell and Kelvin elements

**Maxwell:** spring and dashpot in series

![Maxwell Element Diagram]

**Kelvin:** spring and dashpot in parallel

![Kelvin Element Diagram]
Behaviour of the Maxwell model in creep and relaxation tests

\[ \tau = \frac{\eta}{E} \] is the relaxation time

\[ \frac{\varepsilon}{\varepsilon_0} \] vs. \( t/\tau \)

\[ \frac{\sigma}{\sigma_0} \] vs. \( t/\tau \)

\( \tau = \frac{\eta}{E} \) is the relaxation time
Behaviour of the Maxwell model in a constant strain rate test

Stress-strain curve with three strain-rates $\dot{\varepsilon} = \sigma_r/\eta$, $1.5\sigma_r/\eta$ and $2\sigma_r/\eta$.

$\sigma_r$ is an arbitrary reference stress and $\varepsilon_r = \sigma_r/E$. 
Behaviour of the Kelvin model in creep and relaxation tests

\[ \frac{E\epsilon}{\sigma_0} \quad \frac{\sigma}{\sigma_0} \]

\[ \tau = \frac{\eta}{E} \text{ is the relaxation time} \]
Behaviour of the linear viscoelastic standard solid

\[ \tau = \frac{\eta}{(E_1 + E_2)} \] is the relaxation time
Generalizations

\[(a)\]

\[(b)\]
Creep of metals under stress means time dependent permanent deformation.

Creep is significant at high temperatures when $T > 0.3T_m$, where $T_m$ is the melting temperature in absolute scale.

I primary creep, II secondary creep = steady-state creep, III tertiary creep
Application areas

Important in the analysis of engines, power plant boilers & superheaters etc.

Figures by Valmet Technologies Oy
Deformation mechanism maps

1. Basic Assumptions and Motivation

In creep damage, it is helpful to use a compact method of representation, partly developed by Graham and Walles and later called by Frost and Ashby the "deformation-mechanism map". Schematic illustration of typical map is shown in Fig. 1.6 in which the stress- and temperature-dependent regimes over which different types of creep processes dominate can be captured. Contours of constant strain rates are presented as functions of the normalized equivalent stress $\sigma_{eq}/G$ and the homologous temperature $T/T_m$, where $G$ is the shear modulus and $T_m$ is the melting temperature. For a given combination of the stress $\sigma$ and the temperature $T$, the map provides the dominant creep mechanism and the strain rate $\dot{\varepsilon}$. It shows the range of stress and temperature in which we expect to find each sort of deformation and the strain rate that any combination of them produces (the contours).

The first global overview of the deformation mechanism maps was provided by Frost and Ashby. Later a lot of examples for deformation-mechanism map of different materials were widely presented in literature, refer.e.g. to references. Diagrams like these are available for many metals and ceramics, and are a useful summary of creep behavior, helpful in selecting a material for high-temperature applications.

http://engineering.dartmouth.edu/defmech/
Constitutive model

Decomposition of strain into elastic, creep and thermal parts: \( \varepsilon = \varepsilon^e + \varepsilon^c + \varepsilon^{th} \)

\[
\sigma = E\varepsilon^e = E(\varepsilon - \varepsilon^c - \varepsilon^{th})
\]

Creep strain rate

\( \dot{\varepsilon}^c = f_1(T)f_2(\sigma) \) or \( \dot{\varepsilon}^c = f_1(T)f_2(\sigma, \varepsilon^c, D) \)

Temperature function is of Arrhenius type

\( f_1(T) \sim \exp(-Q/RT) \)

where \( Q \) is the activation energy and \( R \) the gas constant.

Two common choices for the stress dependency

\[
f_2(\sigma) \sim \begin{cases} 
\sigma^p & \text{Norton-Bailey model} \\
\sinh^p \sigma & \text{Garofalo model}
\end{cases}
\]
Norton-Bailey type creep

Creep strain rate in the Norton-Bailey model is

$$\dot{\varepsilon}_c = \frac{1}{t_c} \exp\left(-\frac{Q}{RT}\right) \left(\frac{\sigma}{\sigma_0}\right)^p$$

where $t_c$ is a time parameter, related to the relaxation time and $\sigma_0$ is the drag stress.

NB. The exponent $p$ depends on temperature.
Multiaxial case

Creep strain rate tensor in the Norton-Bailey model is

\[ \dot{\varepsilon}^c = \frac{1}{t_c} \exp(-Q/RT) \left( \frac{\bar{\sigma}}{\sigma_0} \right)^p \frac{\partial \bar{\sigma}}{\partial \sigma} \]

where \( \bar{\sigma} \) is the “effective stress” (scalar).

Different versions of \( \bar{\sigma} \)

\[ \bar{\sigma} = \begin{cases} 
\sigma_{\text{eff}} = \sqrt{3J_2} & \text{von Mises stress} \\
\alpha \sigma_{\text{eff}} + (1 - \alpha) \sigma_1 & \text{convex combination of vM stress and largest principal stress} \\
\alpha \langle \sigma_1 \rangle + \beta I_1 + \gamma \sigma_{\text{eff}} & \text{isochronous form Hayhurst 1972} 
\end{cases} \]

In the isochronous case \( \alpha + \beta + \gamma = 1 \).
Primary and tertiary creep

- Primary creep can be modelled by setting the drag stress dependent on the effective creep strain
  \[
  \varepsilon_{\text{eff}}^c = \int \dot{\varepsilon}_{\text{eff}}^c \, dt, \quad \dot{\varepsilon}_{\text{eff}} = \sqrt{\frac{2}{3}} \varepsilon^c : \dot{\varepsilon}^c
  \]

- Continuum damage mechanics can be used to model tertiary creep
  \[
  \sigma = (1 - D) C^e \varepsilon^e,
  \]
  \[
  \dot{D} = \frac{1}{t_d} \exp\left(-\frac{Q_d}{RT}\right) \left(\frac{\bar{\sigma}}{(1 - D)\sigma_0}\right)^{2r},
  \]

  where \(t_d\) is a time parameter, \(Q_d\) "damage activation energy".

R. Kouhia (Tampere University, Structural Mechanics)
Some empirical rule of thumb relations

- Monkman-Grant (1956) relationship

\[(\dot{\varepsilon}_{\text{min}})^m t_f = C_{MC}\]

- Larson-Miller (1952) parameter \(P\):

\[P_{LM} = T(C + \ln(t_f))\]

where \(C \approx 20\) and fracture time \(t_f\) is given in hours. A recommendable form would be

\[\tilde{P}_{LM} = T \left[ p \ln \left( \frac{\sigma}{\sigma_0} \right) + \ln \left( \frac{t_f}{t_d} \right) \right] = \frac{Q}{R}\]