Introduction to materials modelling

Lecture 2 - Decomposition of stress, geometric interpretation

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Deviatoric stress

Additive decomposition of stress tensor

\[ \sigma_{ij} = s_{ij} + \sigma_m \delta_{ij}, \]

where \( \sigma_m \) is the mean stress and \( s_{ij} \) is the deviatoric stress tensor.

Mean stress is

\[ \sigma_m = \frac{1}{3} \text{tr} \sigma = \frac{1}{3} \sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33} = \sigma_x + \sigma_y + \sigma_z. \]
Deviatoric invariants

Symmetric deviatoric tensor has five independent components.

Eigenvalues of $s$:

$$sn = \lambda n \quad (s - \lambda I)n = 0.$$  

Characteristic equation

$$-\lambda^3 + J_2 \lambda + J_3 = 0,$$

where (notice $J_1 = \text{tr} s = 0$)

$$J_2 = \frac{1}{2} \text{tr}(s^2),$$

$$J_3 = \det s = \frac{1}{3} \text{tr}(s^3).$$
Geometric interpretation

In the principal stress space: hydrostatic axis (blue line), deviatoric plane $\perp$ hydrostatic axis. Red line NP lies on the deviatoric plane.

$$\xi = |\overrightarrow{ON}|, \quad \rho = |\overrightarrow{NP}|, \quad \theta$$

Heigh-Westergaard coordinates