RAK-32320 Introduction to materials modelling

4. exercise – deformation, strain

1. Deformation of a body is described by equations

(a) \( x_1 = X_1, \quad x_2 = X_2 + \alpha X_1, \quad x_3 = X_3, \)

(b) \( x_1 = \sqrt{2\alpha X_1 + \beta}, \quad x_2 = \gamma X_2, \quad x_3 = \delta X_3, \)

(c) \( x_1 = X_1 \cos(\alpha X_3) + X_2 \sin(\alpha X_3), \quad x_2 = -X_1 \sin(\alpha X_3) + X_2 \cos(\alpha X_3), \)
\( \quad x_3 = (1 + \alpha \beta) X_3. \)

Determine deformation gradient \( F \) and the Green-Lagrange strain tensor \( E = \frac{1}{2}(F^T F - I) \). Determine also the infinitesimal strain tensor \( \varepsilon \) and the infinitesimal rotation tensor \( \Omega \).

2. The unit square OABC deforms to a quadrilateral O'A'B'C' in three ways shown below. In each case write down the displacement field \( u_1, u_2 \) as a function of the material coordinates \((X_1, X_2)\). Determine also the deformation gradient \( F \) and the Green-Lagrange strain tensor \( E \). In addition, determine the infinitesimal strain tensor \( \varepsilon \) and the infinitesimal rotation tensor \( \Omega \).

3. With a 120°-strain gauge rosette the following strains are measured: \( \varepsilon_a = 400 \, \mu, \varepsilon_b = 630 \, \mu, \varepsilon_c = -280 \, \mu \). Calculate also the principal strains, the maximum shear strain and their directions.