An anisotropic continuum damage model for concrete

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Introduction

- The non-linear behaviour of quasi-brittle materials under loading is mainly due to damage and micro-cracking rather than plastic deformation.
- Damage of such materials can be modelled using scalar, vector or higher order damage tensors.
- Failure of rock-like materials in tension is mainly due to the growth of the most critical micro-crack.
- Failure of rock-like materials in compression can be seen as a cooperative action of a distributed microcrack array.

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Ottosen’s 4 parameter model

\[ A \frac{J_2}{\sigma_c} + \Lambda \sqrt{J_2} + BI_1 - \sigma_c = 0, \]

\[ \Lambda = \begin{cases} k_1 \cos\left[ \frac{1}{3} \arccos(k_2 \cos 3\theta) \right] & \text{if } \cos 3\theta \geq 0 \\ k_1 \cos\left[ \frac{1}{3} \pi - \frac{1}{3} \arccos(-k_2 \cos 3\theta) \right] & \text{if } \cos 3\theta \leq 0 \end{cases}. \]

\[ \cos 3\theta = \frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}}, : \text{ Lode angle} \]

\( \sigma_c \): the uniaxial compressive strength
\( I_1 = \text{tr} \sigma \): the first invariant of the stress tensor
\( J_2 = \frac{1}{2} s : s, J_3 = \det s = \frac{1}{3} \text{tr} s^3 \): deviatoric invariants
\( A, B, k_1, k_2 \): material constants
**Meridian plane & plane stress**

Green line = Mohr-Coulomb with tension cut-off
Blue line = Ottosen’s model
Red line = Barcelona model
Deviatoric plane

$\pi$ – plane

$\sigma_m = -f_c$

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Thermodynamic formulation

Two potential functions

\[ \psi^c = \psi^c(S) \quad S = (\sigma, D, \kappa) \]

Specific Gibbs free energy

\[ \gamma = \rho_0 \dot{\psi}^c - \dot{\sigma} : \epsilon. \quad \gamma \geq 0. \]

Clausius-Duhem inequality

\[ \varphi(W; S) \quad W = (Y, K) \]

Dissipation potential

\[ \gamma \equiv B_Y : Y + B_K K \]

Define \[ Y = \rho_0 \frac{\partial \psi^c}{\partial D} \quad K = -\rho_0 \frac{\partial \psi^c}{\partial \kappa}, \]

\[ \left( \rho_0 \frac{\partial \psi^c}{\partial \sigma} - \epsilon \right) : \dot{\sigma} + \left( \dot{D} - B_Y \right) : Y + (-\dot{\kappa} - B_K) K = 0. \]

\[ \epsilon = \rho_0 \frac{\partial \psi^c}{\partial \sigma}, \quad \dot{D} = B_Y, \quad \dot{\kappa} = -B_K, \]
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Specific model

Specific Gibbs free energy

\[
\rho_0 \psi^c(\sigma, D, \kappa) = \frac{1 + \nu}{2E} \left[ \text{tr} \sigma^2 + \text{tr}(\sigma^2 D) \right] - \frac{\nu}{2E} \left( 1 + \frac{1}{3} \text{tr} D \right) (\text{tr} \sigma)^2 + \psi^c,\kappa(\kappa)
\]

Elastic domain

\[
\Sigma = \{ (Y, K) | f(Y, K; \sigma) \leq 0 \}
\]

where the damage surface is defined as

\[
f(Y, K; \sigma) = \frac{A\tilde{J}_2}{\sigma_{c0}} + \Lambda \sqrt{\tilde{J}_2} + BI_1 - (\sigma_{c0} + K) = 0,
\]
Invariants in terms of $Y$

\[
\tilde{J}_2 = \frac{1}{1 + \nu} \left[ E \text{tr} Y - \frac{1}{6} (1 - 2\nu)(\text{tr}\sigma)^2 \right]
\]

\[
\tilde{J}_3 = \frac{2}{3(1 + \nu)} \left\{ E [\text{tr}(\sigma Y) - \text{tr}\sigma \text{tr} Y] + \frac{1}{9} (1 - 2\nu)(\text{tr}\sigma)^3 \right\}
\]

\[
\varphi(Y, K; \sigma) = I_\Sigma(Y, K; \sigma)
\]

where $I_\Sigma$ is the indicator function

\[
I_\Sigma(Y, K; \sigma) = \begin{cases} 
0 & \text{if } (Y, K) \in \Sigma \\
+\infty & \text{if } (Y, K) \notin \Sigma
\end{cases}
\]

\[
(B_Y, B_K) = \begin{cases} 
(0, 0), & \text{if } f(Y, K_\alpha; \sigma) < 0, \\
(\dot{\lambda} \frac{\partial f}{\partial Y}, \dot{\lambda} \frac{\partial f}{\partial K}), & \dot{\lambda} \geq 0, \text{ if } f(Y, K_\alpha; \sigma) = 0,
\end{cases}
\]

\[
\dot{D} = \dot{\lambda} \frac{\partial f}{\partial Y}, \quad \dot{\kappa} = -\dot{\lambda} \frac{\partial f}{\partial K}
\]
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Some results

Uniaxial compression - ultimate compressive strength $\sigma_c = 32.8$ MPa

$\sigma_{c0} = 18$ MPa, $\sigma_{t0} = 1$ MPa, $(I_1, \sqrt{J_2}) = (-5\sqrt{3}\sigma_{c0}, 4\sigma_{c0}/\sqrt{2})$

$A = 2.694, B = 5.597, k_1 = 19.083, k_2 = 0.998$

$$K = \frac{a_1(\kappa/\kappa_{\max}) + a_2(\kappa/\kappa_{\max})^2}{[1 + b(\kappa/\kappa_{\max})^2]}$$

$a_1 = 85.3$ MPa, $a_2 = -12.65$ MPa, $b = 0.7032$

Experimental results from Kupfer et al. 1969.
Figure 5: (a) The predicted reduction of the Young's modulus $E$ and apparent Poisson's ratio $\nu_{\text{app}}$ under uniaxial compression. (b) Predicted damage-strain curves of the constitutive model compared to experimental results. 

Figure 6: (a) Comparison between the predicted stress-strain curves for a concrete specimen and the experimental data. (b) Predicted damage-strain curves of the constitutive model compared to experimental data.

Young's modulus and apparent Poisson's ratio

![Graph showing Young's modulus and apparent Poisson's ratio.]

Biaxial compression

![Graph showing biaxial compression behavior.]

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Conclusions and future work

- Continuum damage formulation of the Ottosen’s 4 parameter model
- Can model axial splitting
- Implementation into FE software (own codes, ABAQUS)
- Development of directional hardening model
- Regularization by higher order gradients

Thank you for your attention!