A multiaxial high-cycle transversely isotropic fatigue model

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Outline

- Motivation and background
- Isotropic model
- Transversely isotropic model
- Parameter estimation
- Results
- Conclusions and future developments
Motivation and background

Certain materials exhibit transversely isotropic symmetry

- unidirectionally reinforced composites
- forged metals
  - elasticity isotropic
  - fatigue properties transversely isotropic

Background - Fatigue models

Either stress, strain or energy based.

Stress based criteria are commonly used in high-cycle fatigue

- stress invariant criteria, Sines 1955, Crossland 1956, Fuchs 1979

Cumulative damage theories.

A more fundamental approach using evolution equations.
Continuum approach

Proposed by Ottosen, Stenström and Ristinmaa in 2008.

Endurance surface postulated as

\[ \beta = \frac{1}{\sigma_{oe}} (\bar{\sigma} + AI_1 - \sigma_{oe}), \]

where

\[ \bar{\sigma} = \sqrt{3J_2(s - \alpha)} = \sqrt{\frac{3}{2}(s - \alpha) : (s - \alpha)}, \]

\[ I_1 = \text{tr } \sigma. \]

Back stress and damage evolution eqs.

\[ \dot{\alpha} = C(s - \alpha) \dot{\beta}, \]

\[ \dot{D} = g(\beta, D) \dot{\beta} = K \exp(L \beta) \dot{\beta}. \]
Transversely isotropic model

The stress is decomposed as

$$\sigma = \sigma_L + \sigma_T,$$

where

$$\sigma_T = P \sigma P, \quad P = I - B,$$

where $B = b \otimes b$ is the structural tensor and $b$ is the unit vector normal to the transverse isotropy plane.

Integrity basis of a transversely isotropic solid

$$I_1 = \text{tr } \sigma, \quad I_2 = \frac{1}{2} \text{tr } (\sigma^2), \quad I_3 = \frac{1}{3} \text{tr } (\sigma^3), \quad I_4 = \text{tr } (\sigma B), \quad I_5 = \text{tr } (\sigma^2 B).$$
Endurance surface

Present transversely isotropic formulation

\[
\beta = \left\{ \bar{\sigma} + A_L I_{L1} + A_T I_{T1} - \left[ (1 - \zeta) S_T + \zeta S_L \right] \right\} / S_T = 0,
\]

where

\[
\bar{\sigma} = \sqrt{3J_2(s - \alpha)} , \quad I_{L1} = \text{tr} \sigma_L = I_4, \quad I_{T1} = \text{tr} \sigma_T = I_1 - I_4,
\]

and

\[
\zeta = \left( \frac{\sigma_L : \sigma_L}{\sigma : \sigma} \right)^n = \left( \frac{2I_5 - I_4^2}{2I_2} \right)^n .
\]

In uniaxial loading \( \sigma = \sigma n \otimes n \) the \( \zeta \)-factor has the form

\[
\zeta = (2 \cos^2 \psi - \cos^4 \psi)^n,
\]

where \( \psi \) is the angle between \( n \) and \( b \).
Shape in the $\pi$-plane and $\zeta$-factor

$S_L/S_T = 1$ dotted black line, 1.5 dashed blue line, 2 red line

$A_L = 0.225$, $A_T = 0.275$ $b = (0, 0, 1)^T$
Evolution equations for $\alpha$ and $D$

Damage and the back-stress evolves only when moving away from the endurance surface

$$\dot{D} = \frac{K}{1 - D} \exp(L\beta) \dot{\beta}, \quad \dot{\alpha} = C(s - \alpha) \dot{\beta}.$$
Estimation of the parameters

Five material parameters in the endurance surface $S_L, S_T, A_L, A_T$ and $n$.

Three material parameters in the evolution equations for the back-stress and damage $C, K$ and $L$.

Data from tests with forged 34CrMo6 steel. Due to the lack of data in the intermediate directions we have chosen $n = 1$.

\[ S_L = 447 \text{ MPa}, \quad S_T = 360 \text{ MPa}, \quad A_L = 0.225, \quad A_T = 0.300, \]
\[ C = 33.6, \quad K = 12.8 \cdot 10^{-5}, \quad L = 4.0 \]
Fatigue strengths $\sigma_m = 0$

$\sigma_a$ [MPa]

$N$

$\triangle$ denotes experimental results, ● model predictions
Effect of mean stress

\[ \sigma_x = \sigma_{xm} + \sigma_{xa} \sin(\omega t) \]
longitudinal

\[ \sigma_y = \sigma_{ym} + \sigma_{ya} \sin(\omega t) \]
transverse

\[ \frac{\sigma_{xm}}{\sigma_{xa}} \]

\[ \frac{\sigma_{ym}}{\sigma_{ya}} \]

\( \triangle \) denotes experimental results from McDiarmid 1985 (34CrNiMo6), • model predictions
Effect of mean shear stress

\[ \tau_{xy} = \tau_{xym} + \tau_{xya} \sin(\omega t) \]
**Effect of phase shift**

\[
\begin{align*}
\sigma_x &= \sigma_{xm} + \sigma_{xa} \sin(\omega t) \\
\sigma_y &= \sigma_{xm} + \sigma_{xa} \sin(\omega t - \phi_y) \\
\tau_{xy} &= \frac{1}{2} \sigma_{xa} \sin(\omega t - \phi_{xy})
\end{align*}
\]
Effect of frequency difference

model based on isotropic AISI SAE 4340
exp. results shown 25CrMo4 (Liu & Zenner)
34CrNiMo6 (McDiarmid)

\[ \sigma_x = \sigma_{xa} \sin(\omega_x t) \]
\[ \tau_{xy} = \frac{1}{2} \sigma_{xa} \sin(\omega_{xy} t) \]

\[ \sigma_x = \sigma_{xa} \sin(\omega_{xy} t) \]
\[ \sigma_y = \sigma_{xa} \sin(\omega_{y} t) \]
Conclusions and future developments

- Transversally isotropic continuum based HCF-model
- More tests needed
- Microstructurally based anisotropic damage model
- Constitutive model with anisotropic damage
- Implementation into a FE code

Thank you for your attention!
Deviatoric invariants

Deviatoric invariants and max shear in the longitudinal and in the isotropy plane

\[ J_2 = \frac{1}{2} \text{tr} (s^2), \quad J_4 = \text{tr} (sB), \quad J_5 = \text{tr} (s^2 B). \]

\[ \tau_{\text{max}}(\sigma_T) = \sqrt{J_2 + \frac{1}{4} J_4^2 - J_5}, \quad \tau_{\text{max}}(\sigma_L) = \sqrt{J_5 - J_4^2}. \]