



# A unified LCF-HCF model based on continuum mechanics

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# Introduction - fatigue models

Problems in fatigue analyses:

- Low-cycle- and high-cycle -fatigue regimes are treated separately
- Mostly based on well defined cycles.
- Multiaxiality.

A more fundamental approach for HCF based on *evolution equations* proposed by Ottosen, Stenström and Ristinmaa in IJF 2008.

In this study this idea is combined with a plasticity model to obtain a unified model.



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# Evolution equation based HCF model

Key ingredients are:

## Endurance surface

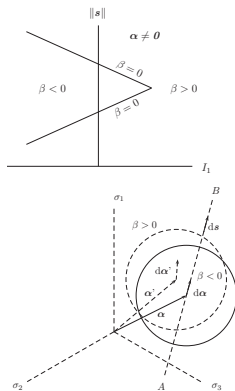
$$\beta(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\}; \text{parameters}) = 0,$$

**evolution equations** for damage  $D$  and the internal variables  $\{\boldsymbol{\alpha}\}$

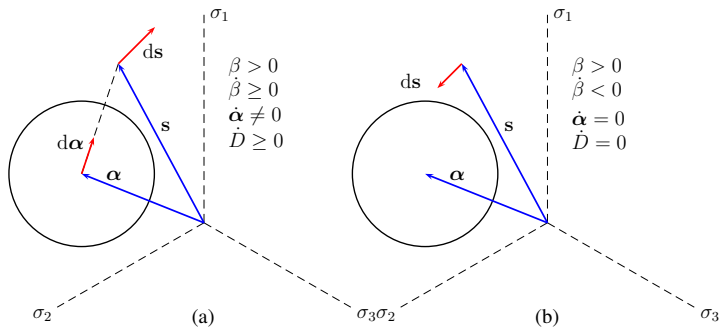
$$\{\dot{\boldsymbol{\alpha}}\} = \{\mathbf{G}\}(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\})\dot{\beta},$$

and

$$\dot{D} = g(\beta, D)\dot{\beta}.$$



# Conditions for evolution



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# Endurance surface

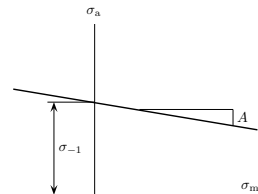
Original formulation by Ottosen et al. for isotropic fatigue

$$\beta = \frac{1}{\sigma_{-1}} \left[ \sqrt{3\bar{J}_2} + AI_1 - \sigma_{-1} \right] = 0,$$

where  $\bar{J}_2 = \frac{1}{2} \text{tr}(\mathbf{s} - \alpha)^2$ ,  $I_1 = \text{tr} \boldsymbol{\sigma}$ ,  $A = \sigma_{-1}/\sigma_0 - 1$ , and

$$\sigma_{-1} = \sigma_{af, R=-1}$$

$$\sigma_0 = \sigma_{af, R=0}$$



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# LCF-HCF approach

Evolution equation for the  $\alpha$ -tensor

$$\dot{\alpha} = C(\mathbf{s} - \alpha)\dot{\beta}$$

and for damage

$$\dot{D} = K \exp[L \exp(-\xi \bar{\epsilon}_p) \beta + M \langle \text{sgn}(f) \rangle \bar{\epsilon}_p] \dot{\beta}$$

Plasticity model based on Armstrong-Frederick model

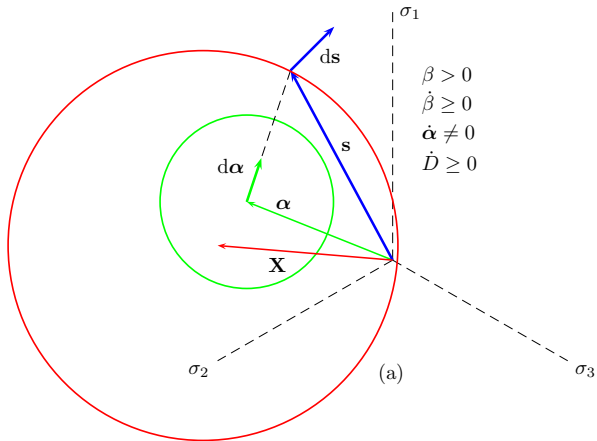
$$f(\boldsymbol{\sigma}, \mathbf{X}, R) = \sqrt{\frac{3}{2}(\mathbf{s} - \mathbf{X}) : (\mathbf{s} - \mathbf{X})} - (\sigma_y + R) = 0$$

$$\dot{R} = \gamma R_\infty (1 - R/R_\infty) \dot{\epsilon}_p$$

$$\dot{\mathbf{X}} = \frac{2}{3} X_\infty \dot{\epsilon}_p - \gamma \dot{\epsilon}_p \mathbf{X}$$

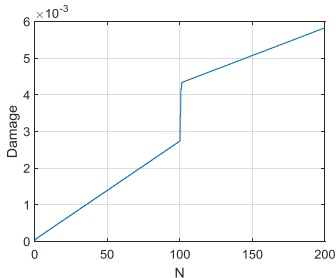
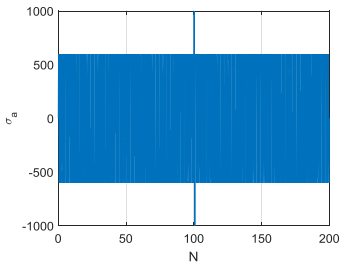
$$\dot{\epsilon}_p = \dot{\lambda} \frac{\partial f}{\partial \boldsymbol{\sigma}}$$

# Illustration in deviatoric plane

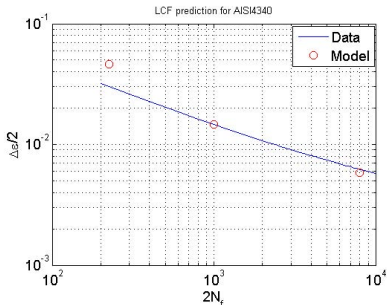


# Some results

## Effect of overstress in fatigue damage evolution



# $\Delta\varepsilon$ -N curve in LCF-regime - AISI 4340



ASTM Handbool (Coffin-Manson + Basquin):

$$\frac{\Delta\varepsilon}{2} = 0.58(2N_f)^{-0.57} + 0.0062(2N_f)^{-0.09}$$

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## Concluding remarks and future work

- Continuum based Unified LCF-HCF model
- Multiaxial, applicable to arbitrary loading history
- *Better plasticity model*
- *Parameter estimation*
- *Micromechanical motivation of the evolution equations.*



Watercolor by  
Pia Erlandsson

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**Thank you for your attention!**

