Modelling of anisotropic fatigue

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Introduction - fatigue models

Either stress, strain or energy based.
Stress based criteria are commonly used in high-cycle fatigue

- stress invariant criteria, Sines 1955, Crossland 1956, Fuchs 1979
- critical plane criteria, Findley 1959, Dang Van 1989, McDiarmid 1990
- average stress criteria, Grübisic and Simburger 1976, Papadopoulos 1997.

Cumulative damage approaches.

A more fundamental approach based on *evolution equations* proposed by Ottosen, Stenström and Ristinmaa in IJF 2008.

Evolution equation based fatigue model

Key ingredients are:

Endurance surface

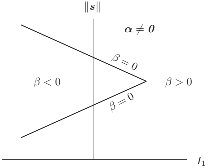
$$\beta(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\}; \text{parameters}) = 0,$$

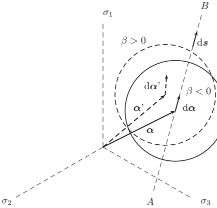
evolution equations for damage D and the internal variables $\{\boldsymbol{\alpha}\}$

$$\{\dot{\alpha}\} = \{\boldsymbol{G}\}(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\})\dot{\beta},$$

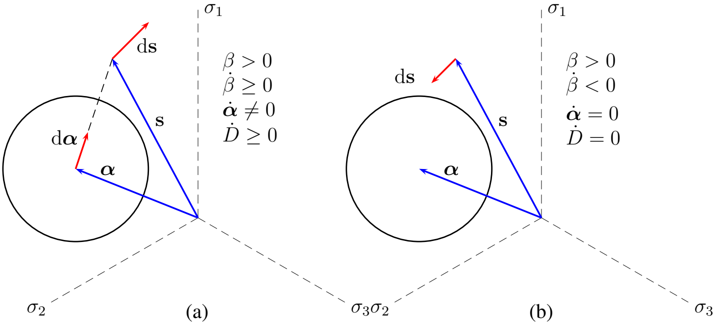
and

$$\dot{D} = g(\beta, D)\dot{\beta}.$$





Conditions for evolution



Endurance surface

Original formulation by Ottosen et al. for isotropic fatigue

$$\beta = \frac{1}{\sigma_{-1}}\left[\sqrt{3\bar{J}_2} + AI_1 - \sigma_{-1}\right] = 0,$$

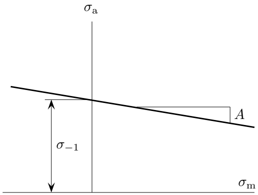
where $\bar{J}_2 = \frac{1}{2}\mathrm{tr}\,(\boldsymbol{s} - \boldsymbol{\alpha})^2, \quad I_1 = \mathrm{tr}\,\boldsymbol{\sigma}, \quad A = \sigma_{-1}/\sigma_0 - 1$, and

$$\sigma_{-1} = \sigma_{\mathrm{af},R=-1}, \quad \sigma_0 = \sigma_{\mathrm{af},R=0},$$

In what follows we use such short hand notation

$$\sigma_{-\mathrm{T}} = \sigma_{\mathrm{T,af},R=-1},$$

$$\sigma_{0\mathrm{T}} = \sigma_{\mathrm{T,af},R=0}, \quad \text{etc.}$$



Endurance surface for transverse isotropy

Simple transversely isotropic endurance surface
(Holopainen et al. EJMA, 2016)

$$\beta = \left\{ \sqrt{3\bar{J}_2} + A_{\mathrm{L}} I_{\mathrm{L1}} + A_{\mathrm{T}} I_{\mathrm{T1}} - \left[(1-\zeta)\sigma_{-\mathrm{T}} + \zeta \sigma_{-\mathrm{L}} \right] \right\} / \sigma_{-\mathrm{T}} = 0$$

where

$$I_{\mathrm{L1}} = \operatorname{tr} \boldsymbol{\sigma}_{\mathrm{L}} = I_4, \quad I_{\mathrm{T1}} = \operatorname{tr} \boldsymbol{\sigma}_{\mathrm{T}} = I_1 - I_4$$

$$\zeta = \left(\frac{\boldsymbol{\sigma}_L : \boldsymbol{\sigma}_L}{\boldsymbol{\sigma} : \boldsymbol{\sigma}} \right)^n = \left(\frac{2I_5 - I_4^2}{2I_2} \right)^n$$

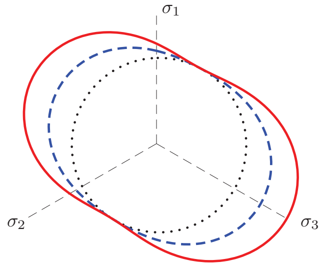
$$I_4 = \operatorname{tr}(\boldsymbol{\sigma} \boldsymbol{M}), \quad I_5 = \operatorname{tr}(\boldsymbol{\sigma}^2 \boldsymbol{M}), \quad \boldsymbol{M} = \boldsymbol{m} \otimes \boldsymbol{m}$$

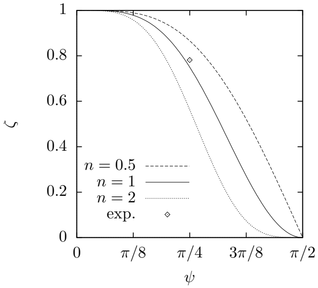
In uniaxial loading $\boldsymbol{\sigma} = \sigma \boldsymbol{n} \otimes \boldsymbol{n}$ the ζ-factor has the form

$$\zeta = (2\cos^2\psi - \cos^4\psi)^n$$

where ψ is the angle between $\boldsymbol{n}$ and $\boldsymbol{m}$.

Shape in the π-plane and ζ-factor





$\sigma_{-\mathrm{L}}/\sigma_{-\mathrm{T}} = 1$ dotted black line, 1.5 dashed blue line, 2 red line
$A_\mathrm{L} = 0.225, A_\mathrm{T} = 0.275$ $\boldsymbol{m} = (0,0,1)^T$

New forms of the endurance surface

Based on reduction of the form similar to the Hill's orthotropic yield criteria

$$\beta = \left(\sqrt{k_1 \bar{J}_4^2 + k_2 \bar{J}_5 + 2k_3 \bar{J}_2} + A_{\mathrm{L}} I_{\mathrm{L1}} + A_{\mathrm{T}} I_{\mathrm{T1}} - \sigma_{-\mathrm{L}} \right) / \sigma_{-\mathrm{L}} = 0$$

where

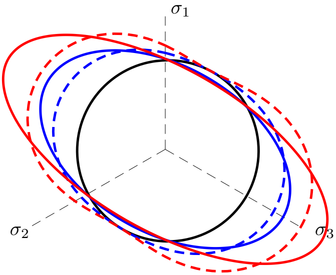
$$\bar{J}_4 = \mathrm{tr}\,[(\boldsymbol{s} - \boldsymbol{\alpha})\boldsymbol{M}], \quad \bar{J}_5 = \mathrm{tr}\,[(\boldsymbol{s} - \boldsymbol{\alpha})^2 \boldsymbol{M}]$$

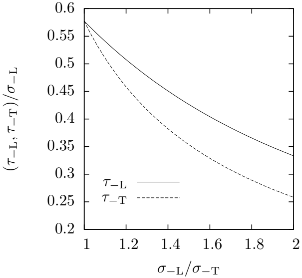
Parameters $k_1, k_2, k_3, A_{\mathrm{L}}$ and A_{T} can be determined from simple fatigue tests as fully reversed and pulsating axial loadings in longitudinal direction and in the isotropy plane + fully reversed torsion in the isotropy plane.

A restricted form can be obtained by constraint $k_3 = \frac{3}{2} - k_2$ and the torsion test is not needed.

Comparison on the π-plane / shear

The restricted transversely isotropic model.



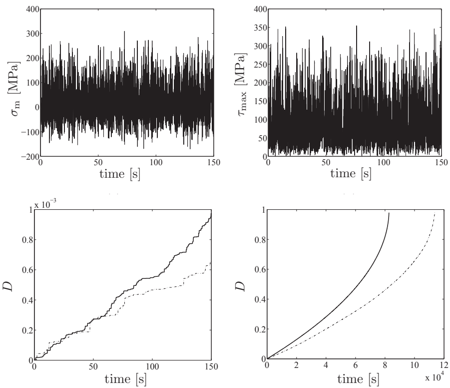


black line isotropic, $\sigma_{-\mathrm{L}}/\sigma_{-\mathrm{T}} = 1.5$ blue line, 2 red line
simple model dashed lines, Hill based model solid lines

Evolution equations

For the internal variable $\boldsymbol{\alpha}$ and damage

$$\dot{\boldsymbol{\alpha}} = C(\boldsymbol{s} - \boldsymbol{\alpha})\dot{\beta}, \qquad \dot{D} = \frac{K}{(1-D)^k} \exp(L\beta)\dot{\beta}$$

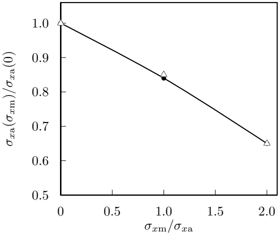


Some results

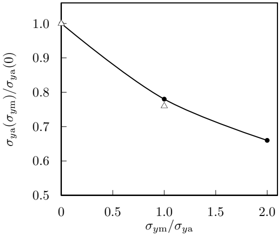
Effect of mean stress

$$\sigma_x = \sigma_{x\mathrm{m}} + \sigma_{x\mathrm{a}} \sin(\omega t) \qquad \sigma_y = \sigma_{y\mathrm{m}} + \sigma_{y\mathrm{a}} \sin(\omega t)$$

longitudinal



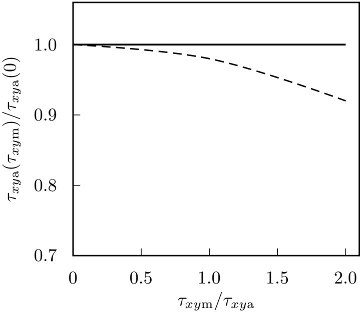
transverse



△ denotes experimental results from McDiarmid 1985 (34CrNiMo6), • model predictions (34CrMo6)

Effect of mean shear stress

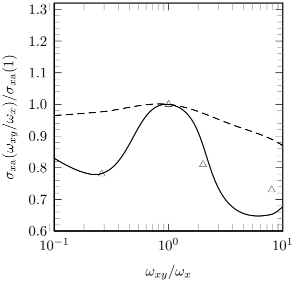
$$\tau_{xy} = \tau_{xy\mathrm{m}} + \tau_{xy\mathrm{a}} \sin(\omega t)$$



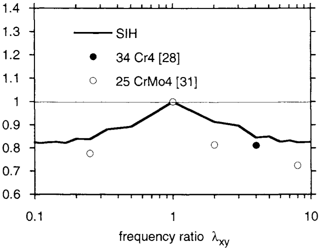
Solid line $N = 10^6$, dashed line $N = 5 \cdot 10^4$.

Effect of frequency difference

$$\sigma_x = \sigma_{xa}\sin(\omega_x t), \quad \tau_{xy} = \tfrac{1}{2}\sigma_{xa}\sin(\omega_{xy} t)$$



present model

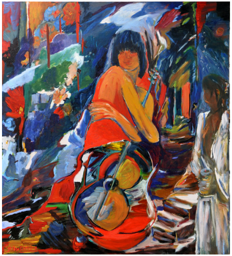


Liu & Zenner

data for isotropic AISI SAE 4340 (dashed line), forged 34CrMo6 (solid line)

Concluding remarks and future work

- Anisotropic, continuum based
- Multiaxial, applicable to arbitrary loading history
- *Parameter estimation - data to the anisotropic fatigue?*
- *Unified LCF-HCF model*
- *Micromechanical motivation of the evolution equations.*



Dream, oil painting by Gisèle L'Épicier

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Thank you for your attention!