Modelling of anisotropic fatigue

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Introduction - fatigue models

Either stress, strain or energy based. Stress based criteria are commonly used in high-cycle fatigue

- stress invariant criteria, Sines 1955, Crossland 1956, Fuchs 1979

Cumulative damage approaches.

A more fundamental approach based on evolution equations proposed by Ottosen, Stenström and Ristinmaa in IJF 2008.
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Evolution equation based fatigue model

Key ingredients are:

**Endurance surface**

\[ \beta(\sigma, \{\alpha\}; \text{parameters}) = 0, \]

**Evolution equations** for damage \( D \) and the internal variables \( \{\alpha\} \)

\[ \{\dot{\alpha}\} = \{G\}(\sigma, \{\alpha\})\dot{\beta}, \]

and

\[ \dot{D} = g(\beta, D)\dot{\beta}. \]
Conditions for evolution

(a) \( \dot{\beta} \geq 0 \) \( \dot{\alpha} \neq 0 \) \( \dot{D} \geq 0 \)

(b) \( \beta > 0 \) \( \dot{\beta} < 0 \) \( \dot{\alpha} = 0 \) \( \dot{D} = 0 \)

Figure 6.9: Ottosen's HCF model. (a) Movement of the endurance surface and damage growth when the stress is outside the endurance surface and moving away from it. (b) When the stress is outside the endurance surface, damage and back stress does not evolve.
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Endurance surface

Original formulation by Ottosen et al. for isotropic fatigue

\[
\beta = \frac{1}{\sigma_{-1}} \left[ \sqrt{3J_2} + AI_1 - \sigma_{-1} \right] = 0,
\]

where \( J_2 = \frac{1}{2} \text{tr} (s - \alpha)^2 \), \( I_1 = \text{tr} \sigma \), \( A = \sigma_{-1}/\sigma_0 - 1 \), and

\[
\sigma_{-1} = \sigma_{af,R=-1}, \quad \sigma_0 = \sigma_{af,R=0},
\]

In what follows we use such short hand notation

\[
\sigma_{-T} = \sigma_{T,af,R=-1},
\]

\[
\sigma_{0T} = \sigma_{T,af,R=0}, \quad \text{etc.}
\]
Endurance surface for transverse isotropy

Simple transversely isotropic endurance surface
(Holopainen et al. EJMA, 2016)

\[ \beta = \left\{ \sqrt{3J_2} + A_L I_{L1} + A_T I_{T1} - [(1 - \zeta)\sigma_T + \zeta\sigma_L] \right\} / \sigma_T = 0 \]

where

\[ I_{L1} = \text{tr} \sigma_L = I_4, \quad I_{T1} = \text{tr} \sigma_T = I_1 - I_4 \]

\[ \zeta = \left( \frac{\sigma_L : \sigma_L}{\sigma : \sigma} \right)^n = \left( \frac{2I_5 - I_4^2}{2I_2} \right)^n \]

\[ I_4 = \text{tr} (\sigma M), \quad I_5 = \text{tr} (\sigma^2 M), \quad M = m \otimes m \]

In uniaxial loading \( \sigma = \sigma n \otimes n \) the \( \zeta \)-factor has the form

\[ \zeta = (2 \cos^2 \psi - \cos^4 \psi)^n \]

where \( \psi \) is the angle between \( n \) and \( m \).
Shape in the $\pi$-plane and $\zeta$-factor

$\sigma_{-L}/\sigma_{-T} = 1$ dotted black line, 1.5 dashed blue line, 2 red line
$A_L = 0.225, A_T = 0.275$ $m = (0, 0, 1)^T$
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New forms of the endurance surface

Based on reduction of the form similar to the Hill’s orthotropic yield criteria

\[
\beta = \left( \sqrt{k_1 \bar{J}_4^2 + k_2 \bar{J}_5 + 2k_3 \bar{J}_2 + A_L I_{L1} + A_T I_{T1} - \sigma_L} \right) / \sigma_L = 0
\]

where

\[
\bar{J}_4 = \text{tr} \left[ (s - \alpha) M \right], \quad \bar{J}_5 = \text{tr} \left[ (s - \alpha)^2 M \right]
\]

Parameters \(k_1, k_2, k_3, A_L\) and \(A_T\) can be determined from simple fatigue tests as fully reversed and pulsating axial loadings in longitudinal direction and in the isotropy plane + fully reversed torsion in the isotropy plane.

A restricted form can be obtained by constraint \(k_3 = \frac{3}{2} - k_2\) and the torsion test is not needed.
Comparison on the $\pi$-plane / shear

The restricted transversely isotropic model.

black line isotropic, $\sigma_{-L}/\sigma_{-T} = 1.5$ blue line, 2 red line
simple model dashed lines, Hill based model solid lines
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Evolution equations

For the internal variable $\alpha$ and damage

\[
\dot{\alpha} = C(s - \alpha)\dot{\beta}, \quad \dot{D} = \frac{K}{(1 - D)^k}\exp(L\beta)\dot{\beta}
\]
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Effect of mean stress

\[ \sigma_x = \sigma_{xm} + \sigma_{xa} \sin(\omega t) \]
\[ \sigma_y = \sigma_{ym} + \sigma_{ya} \sin(\omega t) \]

\( \sigma_{xm} \) = mean stress
\( \sigma_{xa} \) = alternating stress
\( \omega \) = frequency
\( t \) = time

\( \sigma_{ym} \) = mean stress
\( \sigma_{ya} \) = alternating stress

\( \Delta \) denotes experimental results from McDiarmid 1985 (34CrNiMo6), ■ model predictions (34CrMo6)
Effect of mean shear stress

\[ \tau_{xy} = \tau_{xym} + \tau_{xya} \sin(\omega t) \]

![Graph showing the effect of mean shear stress on fatigue life](image)

Solid line \( N = 10^6 \), dashed line \( N = 5 \cdot 10^4 \).
**Effect of frequency difference**

\[ \sigma_x = \sigma_{xa} \sin(\omega_x t), \quad \tau_{xy} = \frac{1}{2} \sigma_{xa} \sin(\omega_{xy} t) \]

The effect of the phase shift corresponds to the case of torsional loading. The minimum of the fatigue limit is situated here.

In the case of uniaxial loading, the fatigue limit of metallic materials can usually be regarded as frequency-independent. In the case of multiaxial loading, however, the frequency difference between the stress components plays an important role. In contrast to the influence of the phase shift, considerably less attention has been paid to the experimental behaviour of the fatigue strength in the presence of differences in frequency of the stress components.

The frequency difference \( \omega_{xy} / \omega_x \) is equal to 0.5, a frequency ratio \( X_{xy} \) of 2 reduces the fatigue limit by about 30 per cent. This behaviour is illustrated in Fig. 9. The plotted curve is not continuous; that is, it applies only to discrete values of the frequency ratio.

In the case of 25CrMo4 and 34CrNiMo6 steel taken from ? and ?, respectively, the data points for 25CrMo4 and 34CrMo6 were used to calculate the points. The markers • and △ represent the test results obtained at a frequency ratio of 0.5 and 1, respectively.

The markers • and △ denote the data points for 25CrMo4 and 34CrMo6, respectively. The x-coordinate direction is parallel with the preferred longitudinal direction. Where \( \omega_x \) is the frequency of the normal stress and \( \omega_{xy} \) is the frequency of the shear stress, the normal stress is higher or lower than that of the shear stress.

The markers • and △ denote the data points for 25CrMo4 and 34CrMo6, respectively. The x-coordinate direction is parallel with the preferred longitudinal direction.

**Fig. 9.** Effect of a frequency difference between a cyclic normal stress and a cyclic shear stress. The solid and dashed line refer to the transverse isotropic and isotropic material, respectively. The markers • and △ denote the data points for 25CrMo4 and 34CrMo6, respectively. (a): Influence of frequency difference on the fatigue strength as the stress state consists of only normal and shear stresses. The solid and dashed line refer to the transverse isotropic and isotropic material, respectively. (b): Influence of frequency difference between two cyclic normal stresses. The markers • and △ denote the data points for 25CrMo4 and 34CrMo6, respectively.
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Concluding remarks and future work

- Anisotropic, continuum based
- Multiaxial, applicable to arbitrary loading history
- *Parameter estimation - data to the anisotropic fatigue?*
- *Unified LCF-HCF model*
- *Micromechanical motivation of the evolution equations.*

Dream, oil painting by Gisèle L’Épicier

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**Thank you for your attention!**