Computational modelling of HCF using a continuum based model

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Content

1 Motivation

2 Endurance surface

3 Model calibration

4 Results

5 Test case - Inclusion problem

6 Conclusions and future developments
1 Motivation

2 Endurance surface

3 Model calibration

4 Results

5 Test case - Inclusion problem

6 Conclusions and future developments
Motivation

- **Classical fatigue models** can be considered as static criteria for alternating stress state and infinite life. For finite life predictions these criterion are augmented by damage accumulation rules based on cycles.

- **Problems:**
  - Complex load histories - cycle counting methods based on well-defined cycles.
  - The effect of loading sequence is not taken into account.

- **Evolution equation based fatigue models** the endurance limit is described with a moving endurance surface.
  - The **state variables** in the endurance surface as well as **damage** are described using **evolution equations**.
  - **Arbitrary loading** histories can be treated in a **unified manner**.
Motivation

Endurance surface

Model calibration

Results

Test case - Inclusion problem

Conclusions and future developments
Endurance surface

Basic idea

The endurance surface is defined in stress space as

$$\beta(\sigma, \alpha; \text{parameters}) = 0,$$

and the evolution of $\alpha$ and damage $D$ is defined as rate-equations

$$\dot{\alpha} = A(\sigma, \alpha) \beta, \quad \dot{D} = g(\beta, D) \beta.$$
Isotropic HCF-model

Proposal by Ottosen, Stenström and Ristinmaa, 2008,

\[
\beta = \frac{1}{S_0} (\bar{\sigma} + AI_1 - S_0) = 0, \quad \beta \geq 0 \quad \text{and} \quad \dot{\beta} > 0,
\]

\[
\bar{\sigma} = \sqrt{3J_2(s - \alpha)} = \sqrt{\frac{3}{2}(s - \alpha):(s - \alpha)}, \quad I_1 = \text{tr } \sigma,
\]

\[
\dot{\alpha} = C(s - \alpha)\dot{\beta}, \quad \dot{D} = K \exp(L\beta)\dot{\beta}.
\]

Meridian plane

Reduces to Haigh-diag. in cyclic loading
Transversely isotropic HCF-model

Certain materials exhibit transversely isotropic symmetry as unidirectional composites or **forged metals**.

Shape of the endurance surface can depend of the invariants

\[ I_1 = \text{tr} \sigma, \quad I_2 = \frac{1}{2} \text{tr} \sigma^2, \quad I_3 = \frac{1}{3} \text{tr} \sigma^3, \quad I_4 = \text{tr} (\sigma B), \quad I_5 = \text{tr} (\sigma^2 B), \]

where \( B \) is the structural tensor \( B = b \otimes b \) and \( b \) is the unit vector normal to the transverse isotropy plane.

**The key idea** in the transversely isotropic model is to split the stress as

\[ \sigma = \sigma_L + \sigma_T, \quad \text{where} \]

\[ \sigma_T = P \sigma P = \sigma - \sigma B - B \sigma + \sigma_b B, \]

and \( P = I - B \) is the projection tensor, \( \sigma_b = I_4 = b \cdot \sigma \cdot b \).
Transversely isotropic endurance surface

Endurance surface for transversely isotropic HC-fatigue model

\[ \beta = \{ \bar{\sigma} + A_L I_{L1} + A_T I_{T1} - [(1 - \zeta) S_T + \zeta S_L] \} / S_T = 0, \]

where

\[ \bar{\sigma} = \sqrt{3J_2(s - \alpha)}, \quad I_{L1} = \text{tr} \sigma_L = I_4, \quad I_{T1} = \text{tr} \sigma_T = I_1 - I_4, \]

and

\[ \zeta = \left( \frac{\sigma_L : \sigma_L}{\sigma : \sigma} \right)^n = \left( \frac{2I_5 - I_4^2}{2I_2} \right)^n. \]

In uniaxial loading \( \sigma = \sigma n \otimes n \) the \( \zeta \)-factor has the form

\[ \zeta = (2 \cos^2 \psi - \cos^4 \psi)^n, \]

where \( \psi \) is the angle between \( n \) and \( b \).
Shape in the $\pi$-plane and $\zeta$-factor

$S_L/S_T = 1$ dotted black line, $1.5$ dashed blue line, $2$ red line
$A_L = 0.225$, $A_T = 0.275$, $b = (0, 0, 1)^T$
Damage evolution

Damage evolution equation modified to

\[
\dot{D} = \frac{K}{(1 - D)^k} \exp(L\beta)\dot{\beta},
\]

where the value \( k = 1 \) has been used.

Complicates sightly the parameter estimation.
1 Motivation

2 Endurance surface

3 Model calibration

4 Results

5 Test case - Inclusion problem

6 Conclusions and future developments
Model calibration

The model is calibrated for two steel grades: forged 34CrMo6 and isotropic AISI-SAE 4340 steel from $R = -1$ tests.

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<tbody>
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<td>34CrMo6</td>
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<td>360</td>
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<td>490</td>
<td>0.225</td>
<td>0.225</td>
<td>0.11</td>
<td>$1.46 \cdot 10^{-5}$</td>
<td>8.7</td>
</tr>
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</table>
1 Motivation

2 Endurance surface

3 Model calibration

4 Results

5 Test case - Inclusion problem

6 Conclusions and *future developments*
Results

The model describes well the mean stress effect in cyclic tension as well as the non-linear effect on mean shear stress on the fatigue strength.

![Graphs showing the effect of mean stress on fatigue life and mean shear stress on fatigue strength.](image)

- **cyclic normal stress in longitudinal and transverse directions**
- **mean shear stress effect on fatigue stress**
**Effect of phase- and frequency difference**

\[
\sigma_x = \sigma_{xm} + \sigma_{xa} \sin(\omega t)
\]
\[
\sigma_y = \sigma_{xm} + \sigma_{xa} \sin(\omega t - \phi_y)
\]
\[
\sigma_{xm} = 1.105\sigma_{xa}, \quad R = 0.05
\]

\[
\sigma_x = \sigma_{xa} \sin(\omega_x t)
\]
\[
\tau_{xy} = \frac{1}{2}\sigma_{xa} \sin(\omega_{xy} t)
\]

Data for isotropic AISI SAE 4340 (dashed line), 34CrMo6 (solid line)
1 Motivation

2 Endurance surface

3 Model calibration

4 Results

5 Test case - Inclusion problem

6 Conclusions and future developments
Test case - Inclusion problem

The model is implemented in Abaqus FE program using the UMAT subroutine. Al$_2$O$_3$ inclusion in a steel plate in plane strain.

Al$_2$O$_3$ inclusion:

$E = 375$ GPa, $\nu = 0.22$

AISI-SAE 4340 steel:

$E = 210$ GPa, $\nu = 0.3$

$S_0 = 490$ MPa, $A = 0.225$, $C = 0.11$

$K = 1.46 \cdot 10^{-5}$, $L = 8.7$
Influence of damage to the behaviour

Damage fields after the cycle 5500 and 8300. Effect of damage taken into account in the constitutive model

Fatigue life $\approx$ 8300 cycles.
1 Motivation

2 Endurance surface

3 Model calibration

4 Results

5 Test case - Inclusion problem

6 Conclusions and future developments
Conclusions and future developments

- An evolution equation based multiaxial transversely isotropic HCF model is developed.
- It can be used for arbitrary loading histories.
- The model is implemented in the Abaqus FE-software using the UMAT subroutine.

- Evolution equation for anisotropic damage,
- Verification of evolution equations from micromechanics, and
- Extension to LCF and high-temperature creep fatigue are under development.
Thank you for your attention!

Amy Winehouse
Acrylic painting by Kelli Gedvil 2013.