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A UNIFIED LCF-HCF MODEL BASED ON CONTINUUM MECHANICS

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Summary. In this work, a unified low- and high-cycle fatigue model based on continuum mechanics is developed. The high-cycle part of the model is based on the concepts of a moving endurance surface in the stress space with an associated evolving isotropic damage variable. The low-cycle part of the model is formulated as a traditional nonlinear isotropic and kinematic hardening J2-plasticity model. The LCF- and HCF-models are connected via the damage evolution equation. Performace of the model is demonstrated with a numerical example.

1 INTRODUCTION

Fatigue of materials under variable loads is a complicated physical process which can even result in catastrophic failure of engineering components. It is characterized by nucleation, coalescence and stable growth of cracks. Nucleation of cracks starts from stress concentrations near persistent slip bands, grain interfaces and inclusions[1,2,3]. Depending on the intensity of loading two ranges of fatigue lives can be classified, namely the low- and high-cycle regime. However, in recent years, it has been observed that fatigue failures can also occur at very high fatigue lives $10^9 - 10^{10}$, below the previously assumed fatigue limits for infinite life.

In high-cycle fatigue, the macroscopic behavior of the material is primarily elastic, while in the low-cycle fatigue regime considerable macroscopic plastic deformations take place. Transition between low- and high-cycle fatigue for metallic materials occurs between $10^3 - 10^4$ cycles.

In this paper a unifield approach to model both low- and high-cycle fatigue of metals is proposed. The high-cycle fatigue part of the model is based on the concept of a moving endurance surface in the stress space with an associated evolving scalar damage variable. In this concept, originally proposed by Ottosen et al.[4] the movement of the endurance surface, as a function of the stress history, is tracked by an evolving back stress type of stress tensor. Therefore this model avoids the ambiguous cycle-counting techniques. A transversally isotropic version of this model is developed by Holopainen et al.[5]. The low-cycle part of the model is formulated by a traditional nonlinear isotropic and kinematic hardening J2-plasticity model. The low and high cycle components are combined so that first, in case of yielding, the stress is returned onto the yield surface and then the damage is computed using a single damage evolution law. In case of elastic behaviour, the only contribution to the damage evolution is due to high-cycle fatigue model. The high-cycle fatigue damage is driven by the amount of violation of the endurance criterion while the low-cycle fatigue is driven in addition by the equivalent plastic strain.

2 MODEL FORMULATION

The continuum fatigue model developed by Ottosen et al.[4] is briefly described. It is based on the assumption that a material exhibit "loading condition dependent endurance limits" within which no damage evolves under cyclic loading. These endurance limits are accomplished by defining a moving endurance surface in stress space. The endurance surface has the form[4]

$$\beta = \frac{1}{\sigma_{-1}}\left[\sqrt{\tfrac{3}{2}(\boldsymbol{s}-\boldsymbol{\alpha}):(\boldsymbol{s}-\boldsymbol{\alpha})} + AI_1 - \sigma_{-1}\right] = 0, \tag{1}$$

where $\boldsymbol{s}$ is the deviatoric stress tensor $\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}I_1\boldsymbol{I}$, $I_1 = \operatorname{tr}\boldsymbol{\sigma}$. The endurance limit at zero mean stress is denoted as σ_{-1} instead of the usual expression $\sigma_{\mathrm{af,R=-1}}$. The non-dimensional positive parameter A is the opposite value of the slope in the Haigh diagram and can be determined e.g. using formula $A = (\sigma_{-1}/\sigma_0) - 1$, where σ_0 is the fatigue limit amplitude for tensile pulsating loading ($R = 0$). A back stress like deviatoric tensor $\boldsymbol{\alpha}$, which memorizes the load history, is responsible for the movement of the endurance surface (1) in the stress space. For the evolution of the α-tensor an evolution rule similar to Ziegler's kinematic hardening rule in plasticity theory is adopted, i.e.

$$\dot{\boldsymbol{\alpha}} = C(\boldsymbol{s}-\boldsymbol{\alpha})\dot{\beta}, \tag{2}$$

where C is a non-dimensional material parameter, and the dot denotes time rate.

In the LCF-region the described HCF-model has to be coupled to the elasto-plastic constitutive model. As in the original formulation by Ottosen et al.[4] a scalar damage variable is chosen to descibe the material deterioration. In the unified LCF-HCF-model the chosen evolution equation for the damage D is

$$\dot{D} = K\exp[L\exp(-\xi\bar{\varepsilon}_{\mathrm{p}})\beta + M\langle\operatorname{sgn}(f)\rangle\bar{\varepsilon}_{\mathrm{p}}]\dot{\beta}, \tag{3}$$

where K, L, ξ and M are material parameters to be estimated from experiments, f is the yield function, $\langle\cdot\rangle$ stands for the Macaulay brackets and $\bar{\varepsilon}_{\mathrm{p}}$ is the equivalent plastic strain.

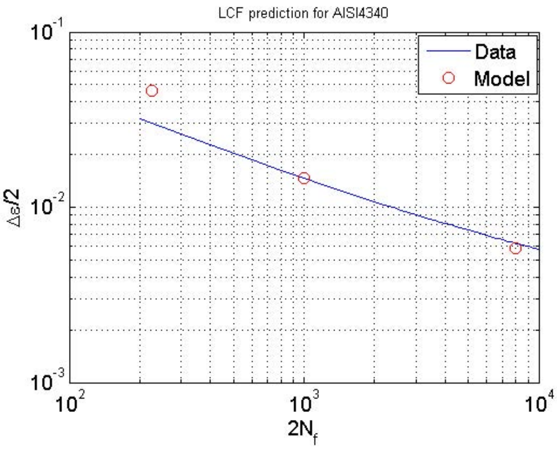

Figure 1: Strain amplitude-fatigue life curve for the AISI 4340 steel.

It should be noticed that damage and the α-tensor only developes when the stress state is moving away from the endurance surface, that is $\beta \geq 0$ and $\dot{\beta} > 0$.

In this preliminary study the constitutive model encompasses the non-linear Armstrong-Frederick isotropic-kinematic hardening J_2-plasticity model coupled with scalar damage

$$f(\boldsymbol{\sigma}, \boldsymbol{X}, R) = \sqrt{\tfrac{3}{2}(\boldsymbol{s} - \boldsymbol{X}) : (\boldsymbol{s} - \boldsymbol{X})} - (\sigma_y + R) = 0, \tag{4}$$

$$\dot{R} = \gamma R_\infty (1 - R/R_\infty)\dot{\bar{\varepsilon}}_p, \tag{5}$$

$$\dot{\boldsymbol{X}} = \tfrac{2}{3} X_\infty \dot{\boldsymbol{\varepsilon}}_p - \gamma \dot{\bar{\varepsilon}}_p \boldsymbol{X}, \tag{6}$$

$$\boldsymbol{\sigma} = (1 - D)\boldsymbol{C}_e(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p), \qquad \dot{\boldsymbol{\varepsilon}}_p = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}}, \tag{7}$$

where $\boldsymbol{X}$ is the back-stress defining the center of the yield surface.

3 EXAMPLE

As an example the SN-curve of the annealed AISI 4340 steel with ultimate tensile strength of 1080 MPa has been computed. The model parameters are $\sigma_{-1} = 490$ MPa, $A = 0.225$, $C = 1.25$, $K = 2.65 \cdot 10^{-5}$, $L = 14.4$, $M = 0.0912$, $\sigma_y = 740$ MPa, $R_\infty = 200$ MPa, $X_\infty = (3/2) \cdot 140$ MPa, $\gamma = 60$ and $\xi = 200$. Comparison to the SN-curve given in the ASTM Handbook[6]

$$\Delta\varepsilon/2 = 0.58(2N_f)^{-0.57} + 0.0062(2N_f)^{-0.09} \tag{8}$$

is given in Fig. 1.

The effect of overstress in the damage evolution is shown in Fig. 2 where there is an overstress cycle of amplitude 1000 MPa inside the basic cyclic loading of amplitude 600 MPa. This overstress cycle will reduce the rate of damage for the subsequent cycles.

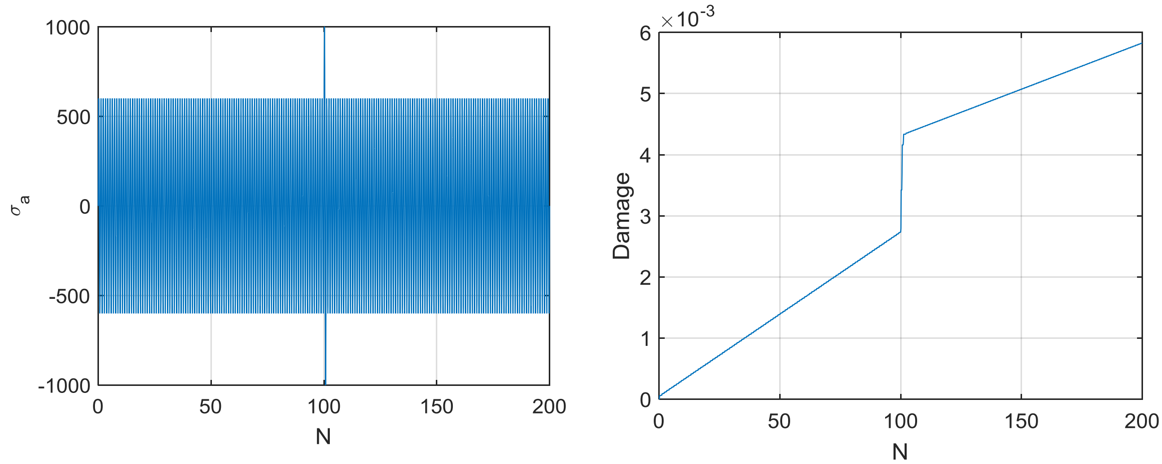

Figure 2: Effect of overstress on damage evolution.

4 CONCLUDING REMARKS

A unified low- and high-cycle fatigue model based on continuum mechanics is developed. The high-cycle part of the model is based on the concepts of a moving endurance surface in the stress space, while the low-cycle part of the model is formulated as the classical J2-plasticity model with nonlinear isotropic and kinematic hardening. Coupling of the LCF- and HCF-models is due to the damage evolution equation. The model parameters have been fitted to the SN-data for the AISI 4340 steel.

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