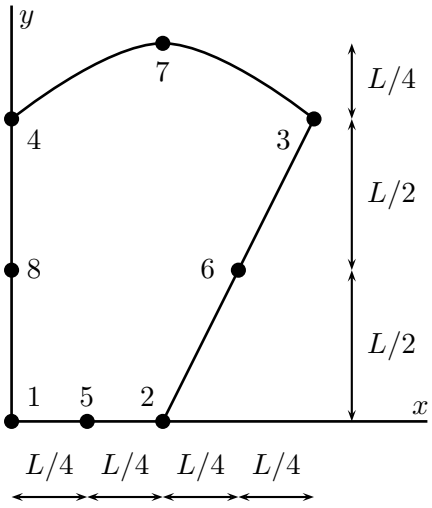
MEI-55200 Numerical methods for field problems

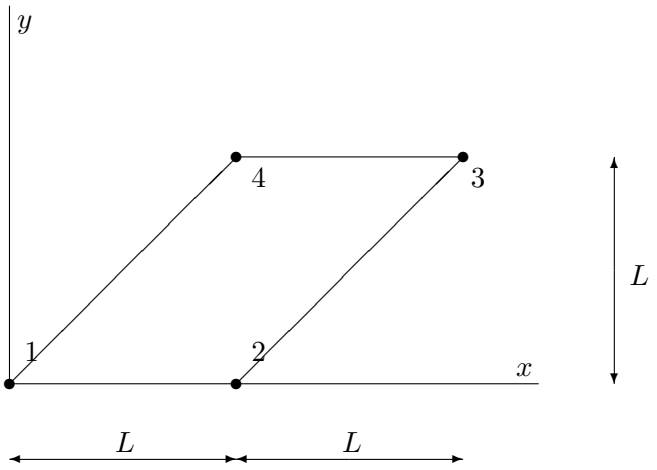
5. Exercise: isoparametric elements

1. Determine the derivative as a function of the global x-coordinate for the following quadratic isoparametric line element. Nodal coordinates are $x_1 = 0, x_2 = \alpha L, x_3 = L$ ($\alpha > 0$). What is the allowable range of the parameter α? The function to be interpolated is $u(x) = u_3(x/L)^2 = \alpha^2 u_3 N_2 + u_3 N_3$, where $N_2 = 1 - \xi^2, N_3 = \frac{1}{2}\xi(1+\xi)$. Draw the derivative du/dx with the following values of the α-parameter: $\alpha = 1/4$ ja $\alpha = 1/3$. What can you say about the accuracy?

2. The nodal temperatures of an isoparametric element shown below are: $u_1 = u_2 = u_5 = 0, u_3 = 2\bar{u}, u_4 = \bar{u}, u_6 = 5/8\bar{u}, u_7 = 35/16\bar{u}, u_8 = 1/2\bar{u}$. Assuming the material to be isotropic with thermal conductivity k, determine the heat flux vector $\vec{q} = -k\nabla u$ at node 4.



Home exercise: For a quadrilateral isoparametric four-node element, the nodal value of the electric potentials are $\phi_1 = \phi_2 = \phi_4 = \phi_0, \phi_3 = -4\phi_0$. Determine the electric field vector $\boldsymbol{E} = -\nabla\phi$ inside the element. The element geometry is shown below.



To be returned at latest in the next exercise!