MEI-55200 Numerical methods for field problems

3. Exercise: FEM in 1-D

1. Solve the 1-D stationary heat transfer problem

$$-(ku')' = 0$$

of a wall $x \in (0, L)$ using FEM. Let's assume that the outside temperature at $x = 0$ is $u_0 > 0$. What is the power needed at $x = L$ to maintain the inside temperature $2u_0$? The conductivity of the wall is defined as

$$k(x) = \begin{cases} 34k_0, & x \in (0, L_1) = (0, \frac{3}{14}L), \quad \text{concrete}, \\ k_0, & x \in (L_1, L_2) = (\frac{3}{14}L, \frac{13}{14}L), \quad \text{glass wool}, \\ 4k_0, & x \in (L_2, L) = (\frac{13}{14}L, L), \quad \text{gypsum}. \end{cases}$$

What is the thermal transmittance (suom. lämmönläpäisykerroin), i.e. the U-value of the wall. The values are $k_0 = 0.05$ W/(mK) and $L = 0.28$ m ($L_1 = 6$ cm, $L_2 = 26$ cm).

2. Solve by FEM the following stationary 1-dimensional diffusion-reaction equation

$$-(ku')' + cu = 0, \quad u(0) = 0, u(L) = \bar{u}_L,$$

where k, c are positive constants $c = \beta^2 k L^{-2}$. Use three equal elements in the domain. Perform computations with the values $\beta = 1$ and 100.

Compute the problem also in the case where the part

$$\int cwudx$$

in the conductivity matrix is lumped. A lumped matrix is obtained as

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \longrightarrow \begin{bmatrix} a_{11} + a_{12} & 0 \\ 0 & a_{21} + a_{22} \end{bmatrix}.$$

What can be concluded?

3. Solve the 1-D stationary heat transfer problem

$$-ku'' = f_0$$

with boundary conditions

$$q(0) = -ku'(0) = -q_0 \quad \text{and} \quad q(L) = -ku'(L) = \alpha q_0.$$

The conductivity k and heat source f_0 are constants ($q_0 = \frac{1}{2} f_0 L$) and α is a positive dimensionless constant. Solve the problem by using a single quadratic element. Does the problem have a solution for arbitrary values of α ($\alpha \geq 0$)? Explain the situation physically.

Home exercise: Solve the fiber pullout problem

$$-E_f A_f \frac{d^2u}{dx^2} + G_m u = 0, \quad u(0) = 0, \quad N(L) = F$$

where the normal force of the fiber is $N = E_f A_f u'$, using the finite element method. The shear modulus of the matrix is assumed to be expressed in the form $G_m = \beta^2 E_f A_f / L^2$, which gives $\beta^2 = G_m L^2 / E_f A_f$.

Design the mesh and choose the element yourself. Try to get the error in displacement less than 1 %.

Draw the results, i.e the solution curves of the displacement u, the axial force N and the reaction force of the matrix $H = G_{\mathrm{m}}u$ in cases where $\beta = 1$ and 10.

To be returned at latest in the next exercise!