MEI-55200 Numerical methods for field problems - exercise 2

2. Exercise: The method of weighted residuals

1. Solve the diffusion-reaction equation with boundary conditions $u(0) = u_0 > 0, u(L) = 0$

$$-k\frac{d^2u}{dx^2} + bu = 0, \quad \text{where} \quad b = \beta^2 k L^{-2}$$

   using a two parametric trial function for temperature u and

   (a) the Galerkin's method,

   (b) the least square method.

   Draw the results with the values of $\beta = 1, 10, 100$.

2. Solve the following beam-column problem:

$$\begin{aligned}
&EI\frac{d^4v}{dx^4} + P\frac{d^2v}{dx^2} = f = \text{constant},\\
&v(0) = v'(0) = 0, \quad M(L) = -EIv''(L) = 0,\\
&Q(L) - Pv'(L) = -EIv'''(L) - Pv'(L) = 0,
\end{aligned}$$

   using the Galerkin method using a two-parametric polynomial trial function. Draw the tip deflection as a function of the compressive load P.

   If the transverse load $f = 0$, the problem is an eigenvalue problem. Solve the eigenvalues P and the corresponding eigenmodes (critical loads, and buckling modes).

Home exercise: Investigate the same problem as in the home exercise 1, i.e. the pull-out-test of a reinforcement fibre. It can be modelled by equation

$$-E_\mathrm{f}A_\mathrm{f}\frac{d^2u}{dx^2} + G_\mathrm{m}u = 0, \qquad u(0) = 0, \quad N(L) = F \tag{1}$$

where the normal force in the fibre is $N = E_\mathrm{f}A_\mathrm{f}u'$. The shear modulus of the matrix is assumed to be expressed in the form $G_\mathrm{m} = \beta^2 E_\mathrm{f}A_\mathrm{f}/L^2$, which gives $\beta^2 = G_\mathrm{m}L^2/E_\mathrm{f}A_\mathrm{f}$.

Solve the problem with the Galerkin method and use a two parametric trial function $u = \alpha_1\phi_1(x) + \alpha_2\phi_2(x)$, using suitable polynomials for the base functions ϕ_1 and ϕ_2.

Draw the results, i.e the solution curves of the displacement u, the axial force N and the reaction force of the matrix $H = G_\mathrm{m}u$ in cases where $\beta = 1$ and 10. Draw also the analytical solution in the same figure.

To be returned at latest in the next exercise!