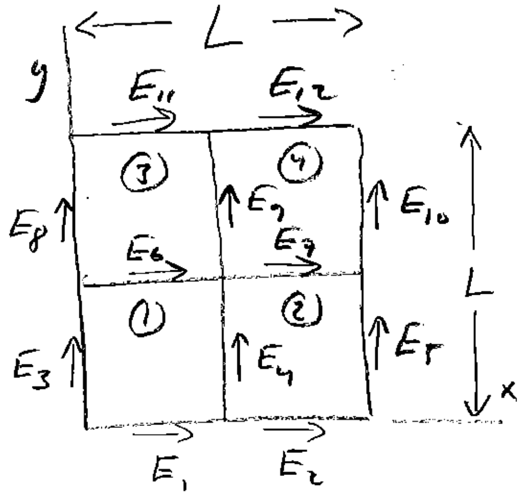
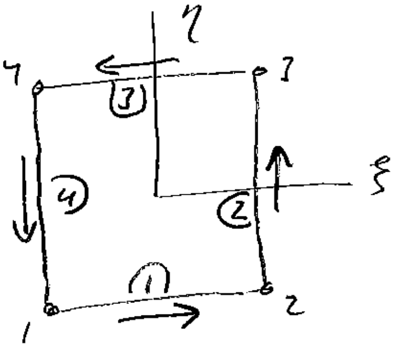


$E_1 = E_2 = E_3 = E_5 = E_8 = E_{10} = E_{11} = E_{12} = 0$

Aktiiviset vapausasteet ovat E_4, E_6, E_7 ja E_9

Suorakaide-elementti paikallisen (ξ,η) koordinaatistossa



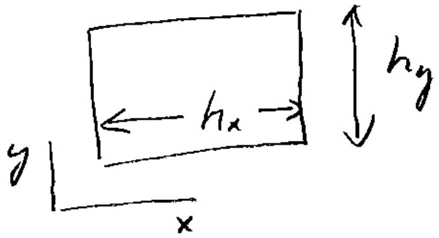
Sivuihin liittyvät interpolaatiofunktiot

$$\underline{N}_1 = \begin{pmatrix} \frac{1}{2}(1-\eta) \\ 0 \end{pmatrix} \quad \underline{N}_2 = \begin{pmatrix} 0 \\ \frac{1}{2}(1+\xi) \end{pmatrix}$$

$$\underline{N}_3 = -\begin{pmatrix} \frac{1}{2}(1+\eta) \\ 0 \end{pmatrix} \quad \underline{N}_4 = -\begin{pmatrix} 0 \\ \frac{1}{2}(1-\xi) \end{pmatrix}$$

Geometriakuvaus on yksinkertainen

$$\underline{J}^T = \begin{bmatrix} \frac{h_x}{2} & 0 \\ 0 & \frac{h_y}{2} \end{bmatrix}$$



Elementin kerroinmatriisi

$$\underline{K}^e = \int_{\Omega^e} \mu^{-1} \underline{B}^T \underline{B} \, dA \qquad \underline{B} = \nabla \times \underline{N}, \quad \underline{N} = [\underline{N}_1 \; \underline{N}_2 \; \underline{N}_3 \; \underline{N}_4]$$

$$\underline{B} = [B_1 \; B_2 \; B_3 \; B_4] \qquad B_i = \frac{\partial N_{yi}}{\partial x} - \frac{\partial N_{xi}}{\partial y} = \frac{2}{h_x}\frac{\partial N_{\eta i}}{\partial \xi} - \frac{2}{h_y}\frac{\partial N_{\xi i}}{\partial \eta}$$

$$\Rightarrow B_1 = \frac{1}{h_y}, \quad B_2 = \frac{1}{h_x}, \quad B_3 = \frac{1}{h_y}, \quad B_4 = \frac{1}{h_x}$$

$$\underline{K}^e = \int_{-1}^{1}\int_{-1}^{1} \mu^{-1} \underline{B}^T \underline{B} \det \underline{J} \, d\xi d\eta = \mu^{-1} \begin{bmatrix} h_x/h_y & 1 & h_x/h_y & 1 \\ & \frac{h_y}{h_x} & 1 & \frac{h_y}{h_x} \\ & & \frac{h_x}{h_y} & 1 \\ \text{Symm.} & & & \frac{h_y}{h_x} \end{bmatrix}$$

Jos $h_x = h_y$

$$\underline{K}^e = \mu^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ & 1 & 1 & 1 \\ \text{Symm.} & & 1 & 1 \\ & & & 1 \end{bmatrix}$$

$$\underline{K}^1 = \mu^{-1} \begin{bmatrix} 1 & 1 & -1 & -1 \\ & 1 & -1 & -1 \\ & & 1 & 1 \\ & & & 1 \end{bmatrix} = K^2 = K^3 = K^4$$

Paikallisten ja globaalien vapausasteiden yhteys

reunaehtovapausasteita ei merkitty.

	1	2	3	4
elem 1		4	6	
2			7	4
3	6	9		
4	7			9

$$\underline{K} = \mu^{-1} \begin{bmatrix} K^1_{22} + K^2_{44} & K^1_{23} & K^2_{43} & 0 \\ & K^1_{33} + K^3_{11} & 0 & K^3_{12} \\ & & K^2_{33} + K^4_{11} & K^4_{14} \\ & & & K^3_{22} + K^4_{44} \end{bmatrix}$$

$$= \mu^{-1} \begin{bmatrix} 2 & -1 & 1 & 0 \\ & 2 & 0 & 1 \\ & & 2 & -1 \\ & & & 2 \end{bmatrix}$$

$$\underline{M}^e_{ij} = \varepsilon \int_{\Omega^e} \underline{N}_i \cdot \underline{N}_i \, d\Omega$$

$$M^e_{11} = \varepsilon \int_{-1}^{1}\int_{-1}^{1} \tfrac{1}{4}(1-\eta)^2 \frac{h_x h_y}{4} \, d\xi d\eta = \varepsilon \frac{h_x h_y}{3} = M^e_{22} = M^e_{33} = M^e_{44}$$

$$M^e_{12} = 0 = M^e_{14} = M^e_{34} = M^e_{23}$$

$$M^e_{13} = \varepsilon \int_{-1}^{1}\int_{-1}^{1} \tfrac{1}{2}(1-\eta)\left[-\tfrac{1}{2}(1+\eta)\right] \frac{h_x h_y}{4} \, d\xi d\eta = -\tfrac{1}{6}\varepsilon h_x h_y$$

$$M_{24} = -\tfrac{1}{6}\varepsilon h_x h_y$$

nyt $h_x = h_y = \frac{L}{2}$

$$\underline{M} = \frac{\varepsilon L^2}{24} \left[\begin{array}{c|c|c|c} 2+2 & 0 & 0 & 0 \\ & 2+2 & 0 & 0 \\ \hline & & 2+2 & 0 \\ & & & 2+2 \end{array}\right] = \frac{\varepsilon L^2}{6} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$\underline{\tilde{K}}\,\underline{\phi} = \omega^2 \varepsilon \mu L^2 \underline{\tilde{M}}\,\underline{\phi} \qquad \underline{\tilde{K}}\,\underline{\phi} = \lambda \underline{\tilde{M}}\,\underline{\phi}$$

$\Rightarrow \lambda = 0, 12, 12$ ja 24

$$\omega = \sqrt{\lambda}\frac{1}{\sqrt{\mu\varepsilon}\,L} = \sqrt{\lambda}\,\frac{c}{L} = \begin{cases} 0 \\ 3{,}464 \frac{c}{L} \\ 3{,}464\, c/L \\ 4{,}899 \frac{c}{L} \end{cases}$$