MEC-7010 Post graduate course
Introduction to higher-order continuum models

3. exercise

1. Using the equations of equilibrium

$$\sigma_{ji,j} + b_i = 0, \tag{1}$$
$$\mu_{ji,j} + \epsilon_{ijk}\sigma_{jk} + c_i = 0, \tag{2}$$

   the constitutive relations

$$\sigma_{ij} = \lambda\gamma_{kk}\delta_{ij} + (\mu + \mu_c)\gamma_{ij} + (\mu - \mu_c)\gamma_{ji}, \tag{3}$$
$$\mu_{ij} = \alpha\kappa_{kk}\delta_{ij} + (\beta + \gamma)\kappa_{ij} + (\beta - \gamma)\kappa_{ji}, \tag{4}$$

   and the kinematical equations

$$\gamma_{ij} = u_{j,i} - \epsilon_{kij}\varphi_k, \qquad \kappa_{ij} = \varphi_{j,i}, \tag{5}$$

   express the equilibrium equations in terms of the displacement and microrotation vectors u_i and φ_i, respectively.

2. By using the ∇-sumbol and the corresponding operators, express the equilibrium equations in the abolute notation in terms of displacement and microrotation vectors $\boldsymbol{u}$ and $\boldsymbol{\varphi}$, respectively.

3. Give the equilibrium equations in the cylindrical coordinate system.

4. Consider an infinite two-dimensional layer of polar continuum of hight h in the x_2-direction. Assume that the displacement and microrotation fields are $\boldsymbol{u} = [u_1(x_2), 0, 0]^T$ and $\boldsymbol{\phi} = [0, 0, \varphi_3(x_2)]^T$ and boundary conditions

$$u_1(0) = \varphi_3(0) = 0, \qquad \boldsymbol{t}(x_2 = h) = \boldsymbol{0}, \qquad \boldsymbol{m}(x_2 = h) = m_{32}\boldsymbol{e}_3, \tag{6}$$

   where $\boldsymbol{e}_3$ is the unit vector in the x_3-direction. Solve the problem, i.e. the displacement and rotation fields and calculate the stress and moment stress fields. Assume the same constitutive equations as in (3) and (4).

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