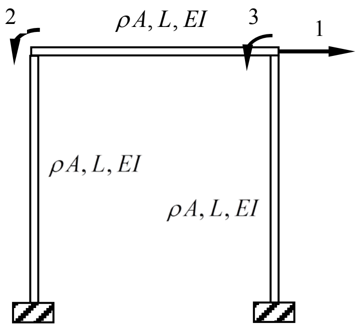


1. Kehän konsistentiksi massamatriisiksi $\mathbf{M}$ ja jäykkyysmatriisiksi $\mathbf{K}$ saatiin

$$\mathbf{M} = \frac{\rho AL}{420}\begin{bmatrix} 732 & 22L & 22L \\ 22L & 8L^2 & -3L^2 \\ 22L & -3L^2 & 8L^2 \end{bmatrix}$$

$$\mathbf{M}^{-1} = \frac{1}{\rho AL^3}\begin{bmatrix} 0,780089L^2 & -3,43239L & -3,43239L \\ -3,43239L & 76,1934 & 38,0116 \\ -3,43239L & 38,0116 & 76,1934 \end{bmatrix}$$

$$\mathbf{K} = \frac{EI}{L^3}\begin{bmatrix} 24 & 6L & 6L \\ 6L & 8L^2 & 2L^2 \\ 6L & 2L^2 & 8L^2 \end{bmatrix}, \quad \mathbf{K}^{-1} = \frac{1}{84EI}\begin{bmatrix} 5L^2 & -3L^2 & -3L^2 \\ -3L^2 & 13L & -L \\ -3L^2 & -L & 13L \end{bmatrix}$$

a) Laske kahden matriisin käänteisellä vektori-iteraatiokaavalla, luentomoniste s. 68, likiarvo alimmalle ominaisarvolle ja ominaisvektorille. Käytä yllä olevaa inverssiä. Paranna ominaisarvon approksimaatiota Rayleigh osamäärällä

b) Valitse uusi ominaisvektoriyrite siten, että se on M-ortogonaalinen alimman ominaisvektorin kanssa ja laske likiarvo toiselle ominaisparille. Paranna ominaisarvon approksimaatiota Rayleigh osamäärällä

c) Laske toinen ominaispari käyttäen origon siirtoa.

We have above stiffness, mass and their inverse matrices for the frame (Ex 1 and 2).

a) Calculate by inverse matrix iteration the approximation for the lowest eigenvalue and eigenvector. Use Rayleigh quotient to get better approximation for the eigenvalue

b) Choose another trial for eigenvector such that it is M-orthogonal for the lowest eigenvector and calculate approximation for the second eigenpair. Use Rayleigh quotient to get better approximation for the eigenvalue.

c) Calculate the second eigenpair using the shift of origin.

a) 1. kierros

alkuarvaus: $\tilde{\phi}_0 = \begin{pmatrix} 1 \\ 1/L \\ 1/L \end{pmatrix}$

$$(K-\lambda M) = \left(\frac{EI}{L^3}\bar{K} - \lambda \rho A L \bar{M}\right) = (\bar{K} - \bar{\lambda}\bar{M})$$

$$\bar{\lambda} = \lambda \frac{\rho A L^4}{EI}$$

$$\tilde{y} = \bar{M}\tilde{\phi} = \begin{pmatrix} 1{,}847619 \\ 0{,}0642857L \\ 0{,}0642857L \end{pmatrix}$$

$$\bar{M} = \frac{1}{420}\begin{bmatrix} 732 & 22L & 22L \\ 22L & 8L^2 & -3L^2 \\ 22L & -3L^2 & 8L^2 \end{bmatrix}$$

$$\bar{K}\tilde{v} = \tilde{y} \overset{\text{ratk.}}{\Rightarrow} \tilde{v} = \begin{pmatrix} 0{,}10538549 \\ -0{,}0568027/L \\ -0{,}0568027/L \end{pmatrix}$$

$$\bar{K} = LL^T \Rightarrow L\underline{z} = \tilde{y} \Rightarrow \underline{z} = \ldots \quad L^T\tilde{v} = \underline{z} \Rightarrow \tilde{v} = \ldots$$

$$\lambda_{tarkka} = 10{,}30684172988$$

$$\tilde{w} = M\tilde{v} = \begin{pmatrix} 0{,}17772109 \\ 0{,}00484397L \\ 0{,}00484397L \end{pmatrix}$$

$$\tilde{\lambda} = \frac{\tilde{v}^T\tilde{y}}{\tilde{v}^T\tilde{w}} = \frac{0{,}187409}{0{,}0181789} = 10{,}3091 \geq \lambda_{tarkka}$$

$$\tilde{y} = \frac{\tilde{w}}{\sqrt{\tilde{v}^T\tilde{w}}} = \begin{pmatrix} 1{,}3181198 \\ 0{,}0359267L \\ 0{,}0359267L \end{pmatrix}$$

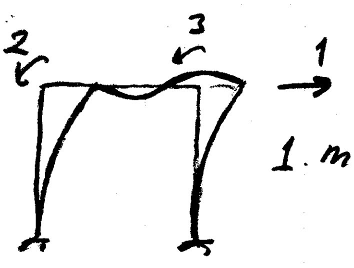
2. kierros $L=1$

$$\tilde{v} = \begin{pmatrix} 0{,}0758333 \\ -0{,}0419433 \\ -0{,}0419433 \end{pmatrix} \qquad \tilde{w} = \begin{pmatrix} 0{,}127877 \\ 0{,}0034761 \\ 0{,}0034761 \end{pmatrix}$$

$$\tilde{\lambda} = \frac{\tilde{v}^T\tilde{y}}{\tilde{v}^T\tilde{w}} = \frac{0{,}097022722}{0{,}00941342} = \underline{10{,}30684194}$$

$$\tilde{\phi}_1 = \frac{\tilde{v}}{\sqrt{\tilde{v}^T \underbrace{M\tilde{v}}_{\tilde{w}}}} = \begin{pmatrix} 0{,}78222 \\ -0{,}43230/L \\ -0{,}43230/L \end{pmatrix}$$

$$K^{-1} = \begin{bmatrix} 0{,}0595238 & -0{,}03571429 & -0{,}03571429 \\ \text{sym} & 0{,}1547619 & -0{,}011904762 \\ & & 0{,}1547619 0 \end{bmatrix}$$



1. muoto

$$\tilde{\vartheta} = K^{-1}y - \frac{1}{\tilde{\lambda}_1}\tilde{\phi}_1\tilde{\phi}_1^T\tilde{y}$$

$$\tilde{\lambda}_1 = 10{,}3068417298$$

$$\tilde{\phi}_1 = \begin{pmatrix} 0{,}7822227136 \\ -0{,}43241093 6/L \\ -0{,}43241093 6/L \end{pmatrix}$$

$$\tilde{\lambda}_1 = 10{,}3068 \frac{EI}{SAL^4}$$

$$\tilde{\omega}_1 = 3{,}2104\sqrt{\frac{EI}{SAL^4}}$$

M-orthgonal trial

yrite

b) $L = 1$ $\quad \tilde{\phi}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \qquad \tilde{y} = M\tilde{\phi}_2$

$$\tilde{y} = \begin{pmatrix} 1{,}74286 \\ 0{,}026190 \\ 0{,}07857 \end{pmatrix} \qquad \tilde{\vartheta}_v = \begin{pmatrix} 0{,}1 \\ -0{,}05912693 \\ -0{,}0503968 \end{pmatrix} \qquad K\tilde{\vartheta}_v = \tilde{y}$$

$$\tilde{\vartheta} = K^{-1}\tilde{y} - \frac{1}{\omega_1^2}\tilde{\phi}_1\tilde{\phi}_1^T\tilde{y} \qquad \text{nyt } \tilde{\vartheta} \perp_M \phi_1$$

$$\tilde{\vartheta} = \begin{pmatrix} -0{,}000029008 \\ -0{,}00383149 \\ 0{,}00489867 \end{pmatrix} \qquad w = \begin{pmatrix} 0{,}005343 \\ -0{,}1094907 \\ 0{,}119156 \end{pmatrix} 10^{-3}$$

$$\tilde{\lambda}_2 = 233{,}2751 \approx \lambda_2^{\text{tarkka}} \quad \left(\lambda_2^{\text{tarkka}} = 229{,}0509\ldots\right)$$

2 kierros

$$\tilde{y} = \begin{pmatrix} 0{,}00533 \\ -0{,}109323 \\ 0{,}118974 \end{pmatrix} \quad \left(\tilde{y} = \frac{w}{\sqrt{\tilde{\vartheta}^T w}}\right) \qquad \tilde{\vartheta} = K^{-1}\tilde{y} - \frac{1}{\omega_1^2}\tilde{\phi}_1\tilde{\phi}_1^T\tilde{y}$$

$$\tilde{\vartheta} = \begin{pmatrix} -0{,}00002711 \\ -0{,}0185259 \\ 0{,}0195236 \end{pmatrix} \qquad w = \begin{pmatrix} 0{,}0049348 \\ -0{,}49375 \\ 0{,}50276 \end{pmatrix} \cdot 10^{-3}$$

$$\lambda = 229{,}284$$

$$\tilde{y} = \begin{pmatrix} 0{,}001147 \\ -0{,}11338377 \\ 0{,}115458 \end{pmatrix} \Rightarrow \tilde{\phi}_2 = \frac{\tilde{\vartheta}}{\sqrt{\tilde{\vartheta}^T M \tilde{\vartheta}}} = \begin{pmatrix} -0{,}0062 \\ -4{,}25428/L \\ 4{,}48334/L \end{pmatrix}$$

$$\left(\tilde{\lambda}_2^{\text{tarkk}} = 229{,}090509 \frac{EI}{SAL^4}\right) \qquad \tilde{\phi}_2^{\text{tarkka}} = \begin{pmatrix} 0 \\ -4{,}3693/L \\ 4{,}3693/L \end{pmatrix}$$

$$\tilde{\omega}_2 = 15{,}142\sqrt{\frac{EI}{SAL^4}}$$

shift of origin

c) origon siirto valitaan $\rho = 200 \frac{EI}{\varrho A L^4} = \bar{\rho} \frac{EI}{\varrho A L^4}$

$$\hat{K} = K - \rho M = \frac{EI}{L^3} \begin{bmatrix} -324,57 & -4,4762L & -4,4762L \\ -4,4762L & 4,1305L^2 & -3,42806L^2 \\ -4,4762L & 3,4286L^2 & 4,1305L^2 \end{bmatrix}$$

$\underline{\underline{L=1}}$

yrite

$$\tilde{\phi} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \qquad \tilde{y} = M\bar{\phi} = \begin{pmatrix} 1,7429 \\ 0,026190 \\ 0,07857 \end{pmatrix} \qquad \bar{\lambda} = \lambda \frac{\varrho A L^4}{EI}$$

$$\tilde{v} = \hat{K}^{-1}\tilde{y} = \begin{pmatrix} -0,0054707 \\ -0,030714 \\ 0,0380597 \end{pmatrix} \qquad (K - \lambda M) = (\bar{K} - \bar{\lambda}\bar{M})$$

$$= [(\bar{K} - \bar{\rho}\bar{M}) - \hat{\lambda}\bar{M}]$$

$$\tilde{w} = M\tilde{y} = \begin{pmatrix} -0,0091511 \\ -0,0011433 \\ 0,00065732 \end{pmatrix} \qquad \hat{\lambda} = \bar{\lambda} - \bar{\rho}$$

$$\hat{\tilde{\lambda}} = \frac{\tilde{v}^T \tilde{y}}{\tilde{v}^T \tilde{w}} = -66,714 \qquad \tilde{y} = \frac{\tilde{w}}{\sqrt{\tilde{v}^T \tilde{w}}} = \begin{pmatrix} -0,87181 \\ -0,108918 \\ 0,06262 \end{pmatrix}$$

2. kierros

$$\tilde{v} = \hat{K}^{-1}\tilde{y} = \begin{pmatrix} 0,002756 \\ -0,114010 \\ 0,111136 \end{pmatrix} \qquad \tilde{w} = M\tilde{y} = \begin{pmatrix} 0,0045999 \\ -0,0028226 \\ 0,003074 \end{pmatrix}$$

$$\hat{\tilde{\lambda}} = \frac{\tilde{v}^T \tilde{y}}{\tilde{v}^T \tilde{w}} = 25,15$$

$$\bar{\lambda}_2 = \hat{\lambda} + \bar{\rho} = 25,15 + 200 = 225,15$$

$$\lambda_2 = \bar{\lambda}_2 \frac{EI}{\varrho A L^4} = 225,15 \frac{EI}{\varrho A L^4} \qquad \lambda_2^{taulukko} = 229,09 \frac{EI}{\varrho A L^4}$$

$$\omega_2 = 14,90 \sqrt{\frac{EI}{\varrho A L^4}} \qquad \phi_2^{taulukko} = \begin{pmatrix} 0 \\ 4,369/L \\ -4,369/L \end{pmatrix}$$

$$\phi_2 = \frac{\tilde{v}}{\sqrt{\tilde{v} M \tilde{v}}} = \begin{pmatrix} 0,1048 \\ -4,385/L \\ -4,274/L \end{pmatrix}$$