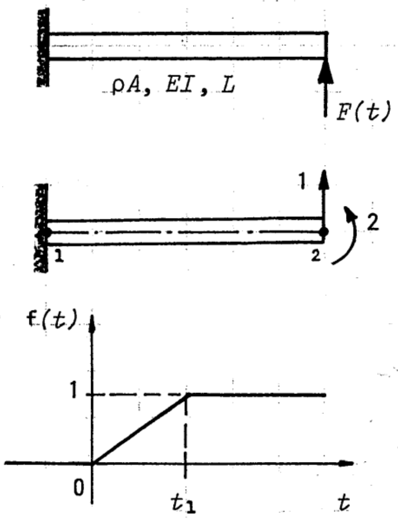
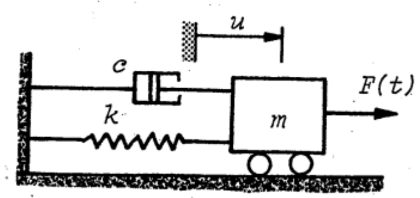


1. Kuvan ulokepalkilla on transienttikuormitus

$$F(t) = F_0 \, \mathrm{f}(t)$$

missä $\mathrm{f}(t)$ on oheisen kuvan mukainen ja $t_1 = \frac{1}{4} T_1$, T_1 on alin ominaisvärähdysaika. Määritä *normaalimuotomenetelmällä* palkin ulokepään siirtymävaste $v(t)$ ja piirrä käyrä aikavälillä $0 \le t \le 2\,t_1$. Palkki on aluksi levossa.

Given transient load determinate the response for $t = 0 \ldots 2t_1$, and $t_1 = T_1/4$ where T_1 is the periodic time for the lowest eigenfrequency.



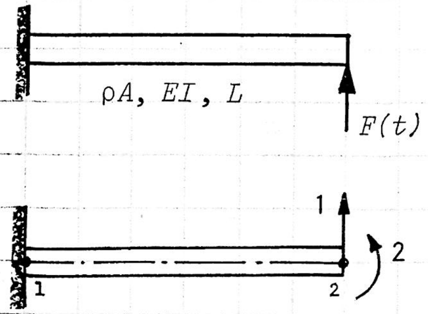
2 Kuvan vaunuun kohdistuu askelheräte

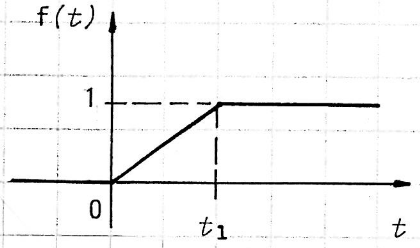
$$F(t) = F_0 \, s(t)$$

Laske aika, jolloin vaunu ensimmäisen kerran saavuttaa ääriasemansa sekä laske vasteen maksimiarvo. Systeemi on alkuhetkellä levossa origossa. $\zeta = 0{,}10$

<u>*Vast*</u>: $u_{max} \approx 1{,}729\, F_0 / k$

The load is a step function. Determinate time when wagon gets its first extreme position and compute the response.





Kuvan ulokepalkilla on transienttikuormitus

$$F(t) = F_0\, f(t)$$

missä $f(t)$ on oheisen kuvan mukainen ja $t_1 = \frac{1}{4} T_1$, T_1 on alin ominaisvärähdysaika. Määritä *normaalimuotomenetelmällä* palkin ulokepään siirtymävaste $v(t)$ ja piirrä käyrä aikavälillä $0 \le t \le 2\,t_1$. Palkki on aluksi levossa.

$$\omega_1 = 3{,}5327\sqrt{\frac{EI}{\rho A L^4}} \quad , \quad \{\phi\}_1 = \begin{bmatrix} 1 \\ 1{,}37744/L \end{bmatrix}$$

$$\omega_2 = 34{,}8069\sqrt{\frac{EI}{\rho A L^4}} \quad , \quad \{\phi\}_2 = \begin{bmatrix} 1 \\ 7{,}62180/L \end{bmatrix}$$

$$[M] = \frac{\rho A L}{420}\begin{bmatrix} 156 & -22L \\ -22L & 4L^2 \end{bmatrix} \Rightarrow \{\phi\}_1^T [M]\{\phi\}_1 = 0{,}24520\ \rho A L$$

$$\{\phi\}_2^T [M]\{\phi\}_2 = 0{,}12621\ \rho A L$$

$$\Rightarrow$$

$$\{\bar\phi\}_1 = \frac{1}{\sqrt{0{,}24520\,\rho A L}}\{\phi\}_1 = \begin{bmatrix} 2{,}0195 \\ 2{,}7817/L \end{bmatrix}\frac{1}{\sqrt{\rho A L}}$$

$$\{\bar\phi\}_2 = \frac{1}{\sqrt{0{,}12621\,\rho A L}}\{\phi\}_2 = \begin{bmatrix} 2{,}81484 \\ 21{,}4542/L \end{bmatrix}\frac{1}{\sqrt{\rho A L}}$$

$$[\bar\Phi] = \frac{1}{\sqrt{\rho A L}}\begin{bmatrix} 2{,}0195 & 2{,}81484 \\ 2{,}7817/L & 21{,}4542/L \end{bmatrix}$$

$$\{\hat R(t)\} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix} \Rightarrow \{Q\} = [\bar\Phi]^T\{\hat R\}$$

$$\Rightarrow \{Q\} = [\bar\Phi]^T\{\hat R\} = \frac{F_0\, f(t)}{\sqrt{\rho A L}}\begin{bmatrix} 2{,}0195 & 2{,}81484 \\ 2{,}7817/L & 21{,}4542/L \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2{,}0195 \\ 2{,}81484 \end{bmatrix}\frac{F_0\, f(t)}{\sqrt{\rho A L}}$$

Pääkoordinaatisto: $\ddot\eta_i + \omega_i^2\,\eta_i = Q_i(t) \quad , \quad i = 1,2$

$$h_i(t-\tau) = \frac{1}{\omega_i}\sin\omega_i(t-\tau)$$

$\underline{0 \le t \le t_1:}$ $\quad f(\tau) = \dfrac{\tau}{t_1}$

$$\eta_i(t) = \frac{Q_i}{\sqrt{\rho A L}}\int_0^t \frac{\tau}{t_1}\,\frac{1}{\omega_i}\sin\omega_i(t-\tau)\,d\tau \qquad \begin{matrix} u = \tau \;,\; u' = 1 \\ v' = \sin \;,\; v = +\frac{1}{\omega_i}\cos \end{matrix}$$

$$= \frac{Q_i}{t_1\,\omega_i^2\sqrt{\rho A L}}\left[\Big/_0^t \tau\cos\omega_i(t-\tau) - \int_0^t \cos\omega_i(t-\tau)\,d\tau\right]$$

(jatkuu)

(jatkoa)

$$\eta_i(t) = \frac{Q_i}{t_1\omega_i^2\sqrt{\rho AL}}\left[t + \frac{1}{\omega_i}\Big/_0^t \sin\omega_i(t-\tau)\right]$$

$$= \frac{Q_i}{t_1\omega_i^3\sqrt{\rho AL}}\left[t\omega_i - \sin\omega_i t\right]$$

$$\{\hat{u}\} = [\bar{\Phi}]\{\eta\} = \frac{1}{\sqrt{\rho AL}}\begin{bmatrix} 2{,}0195 & 2{,}81484 \\ 2{,}7817/L & 21{,}4542/L \end{bmatrix}\begin{bmatrix}\eta_1 \\ \eta_2\end{bmatrix}$$

$$\Rightarrow \quad \{\hat{u}\} = \frac{1}{\sqrt{\rho AL}}\begin{bmatrix} 2{,}0195\,\eta_1 + 2{,}81484\,\eta_2 \\ 2{,}7817\,\eta_1/L + 21{,}4542\,\eta_2/L \end{bmatrix}$$

$$t_1 = \frac{T_1}{4} = \frac{1}{4}\frac{2\pi}{\omega_1} = 0{,}4446\sqrt{\frac{\rho AL^4}{EI}}$$

$$\Rightarrow \quad \eta_1(t) \approx 0{,}10303\,\frac{F_0L^3}{EI}\sqrt{\rho AL}\left[t\omega_1 - \sin\omega_1 t\right]$$

$$\eta_2(t) \approx 0{,}00015014\,\frac{F_0L^3}{EI}\sqrt{\rho AL}\left[t\omega_2 - \sin\omega_2 t\right]$$

$\underline{t \geq t_1:}$ $\quad f(\tau) \equiv 1$

$$\eta_i(t) = \frac{Q_i}{\sqrt{\rho AL}}\left[\int_0^{t_1}\frac{\tau}{t_1}\cdot\frac{1}{\omega_i}\sin\omega_i(t-\tau)\,d\tau + \int_{t_1}^{t}\frac{1}{\omega_i}\sin\omega_i(t-\tau)\,d\tau\right]$$

$$= \frac{Q_i}{\omega_i^2 t_1\sqrt{\rho AL}}\left[\Big/_0^{t_1}\tau\cos\omega_i(t-\tau) - \int_0^{t_1}\cos\omega_i(t-\tau)\,d\tau + t_1\Big/_{t_1}^{t}\cos\omega_i(t-\tau)\right]$$

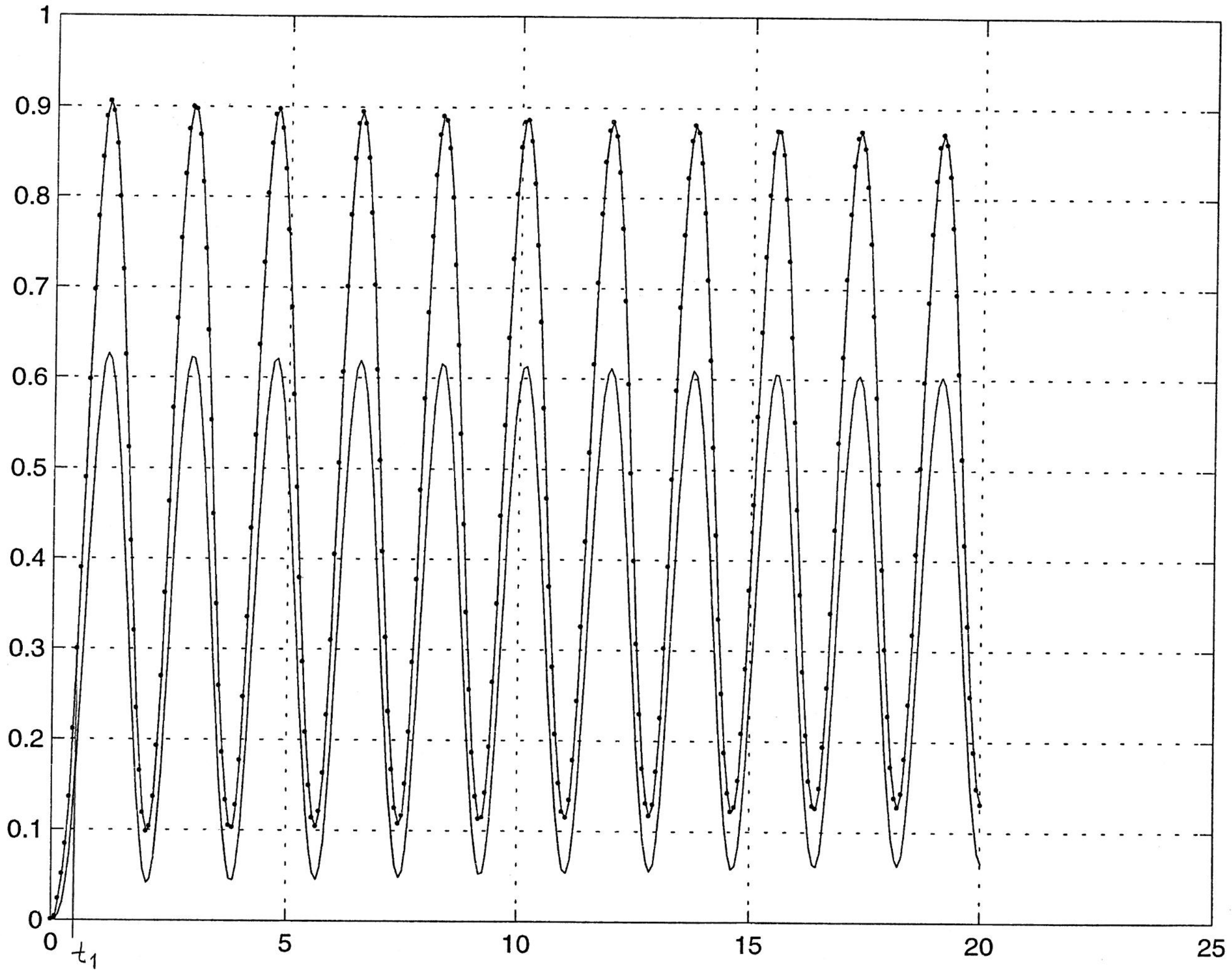
$$= \frac{Q_i}{t_1\omega_i^3\sqrt{\rho AL}}\left[t_1\omega_i \cancel{\cos}(t-t_1) + \Big/_0^{t_1}\sin\omega_i(t-\tau) + t_1\omega_i(1-\cos\omega_i\cancel{(t-t_1)})\right]$$

$$= \frac{Q_i}{t_1\omega_i^3\sqrt{\rho AL}}\left[t_1\omega_i + \sin\omega_i(t-t_1) - \sin\omega_i t\right]$$

$$\Rightarrow$$

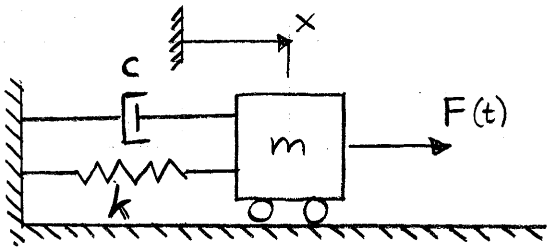
$$\eta_1(t) \approx 0{,}10303\,\frac{F_0L^3}{EI}\sqrt{\rho AL}\left[t_1\omega_1 + \sin\omega_1(t-t_1) - \sin\omega_1 t\right]$$

$$\eta_2(t) \approx 0{,}00015014\,\frac{F_0L^3}{EI}\sqrt{\rho AL}\left[t_1\omega_2 + \sin\omega_2(t-t_1) - \sin\omega_2 t\right]$$



Normaalimuotomenetelmä

③



askelheräte $F(t) = F_0\, s(t)$

$\xi = 0{,}10$

vaimennetun värähtelijän askelvaste

$$x(t) = x_{st}\left[1 - e^{-\xi\omega t}\left(\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d t) + \cos(\omega_d t)\right)\right], \quad t \geq 0$$

$$\omega_d = \omega\sqrt{1-\xi^2}, \quad \omega = \sqrt{\frac{k}{m}}, \quad x_{st} = \frac{F_0}{k}$$

Vasteen maksimiarvo saadaan ajanhetkellä t_p, jolloin $\dot{x}(t_p) = 0$

$$\dot{x}(t) = x_{st}\left[-e^{-\xi\omega t}\cdot(-\xi\omega)\left(\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d t) + \cos(\omega_d t)\right) + \right.$$

$$\left. - e^{-\xi\omega t}\left(\frac{\xi}{\sqrt{1-\xi^2}}\cos(\omega_d t)\cdot\omega_d - \sin(\omega_d t)\cdot\omega_d\right)\right]$$

$$\dot{x}(t_p) = x_{st}\left[\xi\omega\left(\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d t_p) + \cos(\omega_d t_p)\right) + \right.$$

$$\left. - \left(\frac{\xi\omega_d}{\sqrt{1-\xi^2}}\cos(\omega_d t_p) + \omega_d \sin(\omega_d t_p)\right)\right]e^{-\xi\omega t} = 0$$

$$\Rightarrow \xi\omega\left(\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d t_p) + \cos(\omega_d t_p)\right) - \xi\omega\cos(\omega_d t_p) + \omega_d \sin(\omega_d t_p) = 0$$

$$\Rightarrow \left(\frac{\xi^2\omega}{\sqrt{1-\xi^2}} + \omega_d\right)\sin(\omega_d t_p) = 0$$

$$\Rightarrow \frac{\xi^2 + 1 - \xi^2}{\sqrt{1-\xi^2}}\,\omega\sin(\omega_d t_p) = 0 \Rightarrow \sin(\omega_d t_p) = 0 \Rightarrow \omega_d t_p = \pi$$

$$\Rightarrow t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega\sqrt{1-\xi^2}}$$

$$X_{max} = x(t_p) = x_{st}\left[1 - e^{-\xi\omega t_p}(0 + (-1))\right]$$

$$= x_{st}\left[1 + e^{-\xi\pi/\sqrt{1-\xi^2}}\right] \approx 1{,}729\,\frac{F_0}{k}$$