

KAAVAKOKOELMA FEM 2013S II 8.11.13)

$$N_1(\xi) = \frac{1}{2}(1-\xi) \quad N_2(\xi) = \frac{1}{2}(1+\xi) \quad \xi \in [-1, 1]$$

$$\left\{ \begin{array}{l} N_1(\xi, \eta) = N_1 = \frac{1}{4}(1-\xi)(1-\eta) \\ N_2(\xi, \eta) = N_2 = \frac{1}{4}(1+\xi)(1-\eta) \\ N_3(\xi, \eta) = N_3 = \frac{1}{4}(1+\xi)(1+\eta) \\ N_4(\xi, \eta) = N_4 = \frac{1}{4}(1-\xi)(1+\eta) \end{array} \right. \quad \mathbf{J} = \frac{1}{4} \begin{bmatrix} (1-\eta)x_{21} + (1+\eta)x_{34} & (1-\eta)y_{21} + (1+\eta)y_{34} \\ (1-\xi)x_{41} + (1+\xi)x_{32} & (1-\xi)y_{41} + (1+\xi)y_{32} \end{bmatrix}$$

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{Q}_0 = \mathbf{0}$$

$$\mathbf{f}^p = \iint_A \mathbf{N}^T \mathbf{p} dA$$

$$\mathbf{f}^V = \iiint_V \mathbf{N}^T \mathbf{f} dV \quad \mathbf{F} = \mathbf{P} + \sum_e (\mathbf{f}_e^V + \mathbf{f}_e^p)$$

$$\mathbf{K} = \sum_e \mathbf{k}_e \quad \mathbf{M} = \sum_e \mathbf{m}_e$$

$$N_1^s = -\frac{1}{2}\xi(1-\xi) \quad \mathbf{u} = \begin{bmatrix} u \\ v \end{bmatrix} = \mathbf{N} \mathbf{q}$$

$$N_2^s = \frac{1}{2}\xi(1+\xi)$$

$$N_3^s = 1 - \xi^2$$

$$\begin{bmatrix} u_{,\xi} \\ u_{,\eta} \end{bmatrix} = \begin{bmatrix} x_{,\xi} & y_{,\xi} \\ x_{,\eta} & y_{,\eta} \end{bmatrix} \begin{bmatrix} u_{,x} \\ u_{,y} \end{bmatrix} = \mathbf{J} \begin{bmatrix} u_{,x} \\ u_{,y} \end{bmatrix}$$

n	ξ_i	w_i
1	0	2
2	± 0.577350269189626	1
3	0 ± 0.774596669241483	0.888888888888889 0.5555555555555556
4	± 0.339981043584856 ± 0.861136311594053	0.652145158462546 0.347854845137454
5	0 ± 0.538469310105683 ± 0.906179845938664	0.568888888888889 0.478628670499366 0.236926885056189

$$x(\xi, \eta) = N_1(\xi, \eta)x_1 + N_2(\xi, \eta)x_2 + \dots + N_n(\xi, \eta)x_n$$

$$u(\xi, \eta) = N_1(\xi, \eta)q_1 + N_2(\xi, \eta)q_3 + \dots + N_n(\xi, \eta)q_{2n-1}$$

$$v(\xi, \eta) = N_1(\xi, \eta)q_2 + N_2(\xi, \eta)q_4 + \dots + N_n(\xi, \eta)q_{2n}$$

$$J_x = N_{1,\xi}x_1 + N_{2,\xi}x_2N_{3,\xi}x_3$$

$$J_y = N_{1,\xi}y_1 + N_{2,\xi}y_2N_{3,\xi}y_3$$

$$r(\xi, \eta) = N_1r_1 + N_2r_2 + N_3r_3 + N_4r_4$$

$$\mathbf{m}_e = \frac{\rho Al_e}{420} \begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 156 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix} \quad \mathbf{m}_{ek} = \frac{\rho Al_e}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{m}_e = \frac{\rho Al_e}{6} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \quad \mathbf{m}_{ek} = \frac{\rho Al_e}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{k}_e = \frac{EA}{l_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

$$\mathbf{I} \approx \sum_{i=1}^n \sum_{j=1}^n w_i w_j \mathbf{f} \left[\mathbf{x}(\xi_i, \eta_j) \right] \det \mathbf{J}(\xi_i, \eta_j)$$

$$\mathbf{k}_e = \frac{EI_z}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$