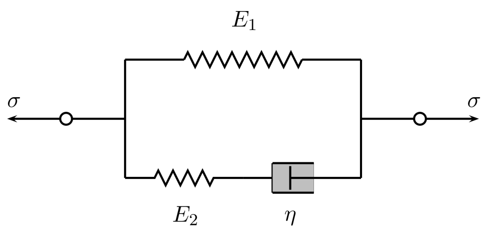
Introduction to materials modelling

11. exercise – viscoelasticity

1. Derive the constitutive equation for the standard linear viscoelastic solid model shown below. Determine also the creep compliance and relaxation modulus and the relaxation time.



2. The Maxwell model combines the linear elastic spring and linear viscous dashpot in series. The constitutive model for the linear spring is $\sigma = E\varepsilon$ ja for the linear viscous dashpot $\dot{\varepsilon} = \sigma/\eta$, where E is the Young's modulus, η viscosity and $\dot{\varepsilon}$ is strain rate.

   - Determine the stress response $\sigma(t)$ when the model is loaded by cyclic straining $\varepsilon(t) = \varepsilon_a \sin(\omega t)$, where ε_a is the strain amplitude. What can you say about the response if $\omega \gg 1/\tau_{rel}$, where $\tau_{rel} = \eta/E$ is the relaxation time of the model.

   Hint. Try solution in the form $\sigma(t) = \sigma_a \sin(\omega t + \phi)$. Solve the phase-angle ϕ and stress amplitude σ_a. Some formulas which might be useful:

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta,$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta,$$
$$\cos\alpha = \frac{1}{\sqrt{1 + \tan^2\alpha}}.$$

3. Determine the central point deflection $v(L/2, t)$ of a simply supported beam made of Kelvin-Voigt material. Draw the time history of the deflection. The loading is shown on the rhs figure.