Introduction to materials modelling

10. exercise – plastic models, continuum damage models

1. Gurson presented in 1977 a yield condition for a porous medium[1]

$$f(I_1, J_2) = \frac{3J_2}{\sigma_\mathrm{y}^2} - \left[1 + \xi^2 - 2\xi \cosh\left(\frac{I_1}{2\sigma_\mathrm{y}}\right)\right],$$

   where ξ is the volumetric pore fraction and σ_y is the yield strength of the metal matrix.

   (a) Sketch the locus of yield surface in the meridian plane $(I_1, \sqrt{3J_2})$ for some values of xi (e.g. $\xi = 0.1$).

   (b) Determine the expression for the plastic strain rate $\dot{\varepsilon}_{ij}^\mathrm{p}$ when assuming associative flow rule $\dot{\varepsilon}_{ij} = \dot{\lambda}\partial f/\partial\sigma_{ij}$.

   The volumetric prore fraction ξ can be considered as a damage variable, having own evolution equation, which consists of growth and nucleation of pores

$$\dot{\xi} = \dot{\xi}_\mathrm{growth} + \dot{\xi}_\mathrm{nucl} = (1-\xi)\mathrm{tr}(\dot{\boldsymbol{\varepsilon}}^\mathrm{p}) + \frac{\xi_N}{S_N\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\bar{\varepsilon}^\mathrm{p} - \varepsilon_N}{S_N}\right)\right]\dot{\bar{\varepsilon}}^\mathrm{p},$$

   where $\bar{\varepsilon}^\mathrm{p}$ the equivalent plastic strain, S_N, ε_N and ξ_N are model parameters.

2. Compare two different continuum damage representations in uniaxial case

$$\sigma = (1-D)E\varepsilon^\mathrm{e}, \tag{1}$$
$$\sigma = \exp(-D)E\varepsilon^\mathrm{e}, \tag{2}$$

   assuming that both damage descriptions have the same evolution equation

$$\dot{D} = \frac{1}{t_\mathrm{d}}\left(\frac{\varepsilon^\mathrm{e}}{\varepsilon_\mathrm{r}}\right)^{2r}, \tag{3}$$

   where t_d and r are material parameters. The reference value for strain can also be expressed as $\varepsilon_\mathrm{r} = \sigma_\mathrm{r}/E$. Notice that in the model (2) the damage variable is not bounded above.

   Solve the response in a constant strain rate tensile test where strain is increasing linearly as $\varepsilon = \dot{\varepsilon}_0 t$, where $\dot{\varepsilon}_0$ is the strain rate which is kept constant in the test. Draw the results in the $(\varepsilon/\varepsilon_\mathrm{r}, \sigma/\sigma_\mathrm{r})$-coordinate system. Assume that there are no inelastic strains, i.e. $\varepsilon = \varepsilon^\mathrm{e}$.

   Determine also the value of the damage variable and strain at the point of maximum stress. Draw the damage as a function of strain. Investigate the effect of r, for example using the values $r = 1, 2, 4$.

[1] A.L. Gurson. Continuum theory of ductile rupture by void nucleation and growth: Part I - Yield criteria and flow rules for porous ductile media. *Journal of Engineering Material and Technology*, 99(1):2–15, 1977. `https://doi.org/10.1115/1.3443401`