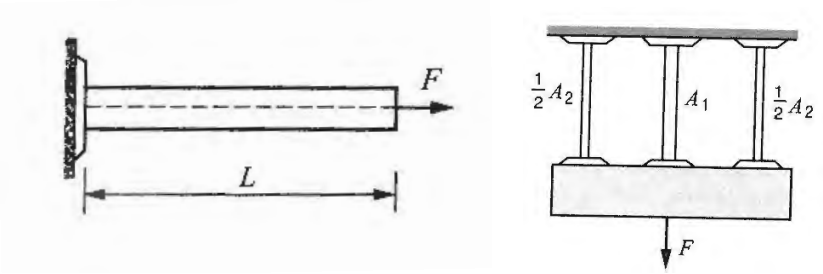
Introduction to materials modelling

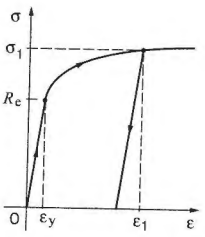
8. exercise – elasto-plastic material model

1. A rod is stretched by a force F until it is elongated by 7 mm. After that poin the load is removed. How large permanent deflection remains in the rod. The material is linearly elastic and ideal plastic with material properties $E = 200$ GPa, $L = 500$ mm, $R_e = 275$ MPa (yield strength) and $A = 60$ mm^2.



2. A ridid plate is supported by three bars. The material of the bars can be modelled by linearly elastic ideally plastic model. All the bars have the same Young's modulus E, but the yield stresses are different: R_{e1} and $R_{e2} > R_{e1}$. The average stress in the bars is defined as $\bar{\sigma} = F/(A_1 + A_2)$.
   (a) Calculate $\bar{\sigma}$, when all bars are yielding.
   (b) When the load is removed determine the permanent strains and the residual stresses in the bars.
   (c) If the loading is continued in the compressive side such that the middle bar is yielding, what is then $\bar{\sigma}$?

3. Assuming that the stress-strain relation in the elasto-plastic range is $\sigma = k\varepsilon^n$ (i.e. when $\sigma > R_e$). Determine the material parameters k and n and the permanent strain when the material is stressed to the level of $\sigma = \sigma_1 = 320$ MPa and then the stress is removed. The yield stress is $R_e = 266$ MPa, $\varepsilon_y = 5800\ \mu$, $\varepsilon_1 = 12000\ \mu$.



4. Thin walled circular tubes are often used for investigations of elasto-pastic material models. Assume that the wall thickness t is much smaller than the radius R of the cross section, i.e. $t/R \ll 1$. The tube is loaded by a twisting moment T and by a normal force N.
   (a) If the material obeys von Mises yield condition $\sqrt{3J_2} = \sigma_y$, where σ_y is the yield stress. Write the yield condition out in terms of the twisting moment T and normal force N. J_2 is the second invariant of the deviatoric stress $J_2 = \frac{1}{2}\mathrm{tr}(\boldsymbol{s}^2)$.
   (b) Determine the Lode angle θ when the loading is only (i) the twisting moment or (ii) the normal force (notice the sign of the normal force)?
   (c) If the normal force N alone produces a stress which is half of the yield stress, how large internal overpressure the tube can carry prior yielding.