Introduction to materials modelling

3. exercise – stress, balance equations

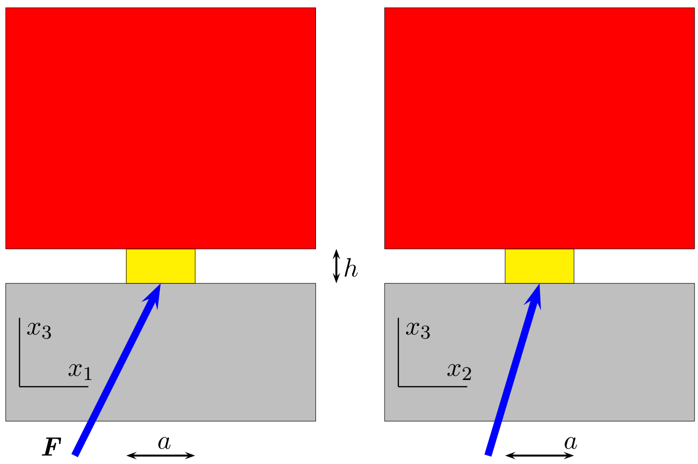
1. As shown in the figure below the measured reaction force vector of a large engine block is $\boldsymbol{F} = (5F, F, 10F)^T$. Dimensions of the bearing plate are $a \times a \times h$. In addition, it is known that the normal stress in the engine block in the direction of x_1 axis is $\sigma_{11} = -6\sigma_0$, where $\sigma_0 = F/a^2$.

   (a) Determine based on the above knowledge as many components of the stress tensor as possible in the (x_1, x_2, x_3)-coordinate system. What elements cannot be betermined based on the above knowledge.

   (b) If it is assumed that the unknown stress components are zero, determine the principal stresses and the normal direction of the largest principal stress. What is the largest shear stress?

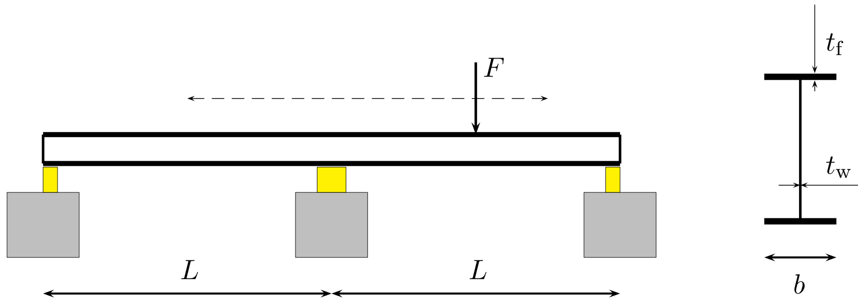
   (c) Determine the stress deviator $\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma})\boldsymbol{I}$ and its second invariant $J_2 = \frac{1}{2}\mathrm{tr}(\boldsymbol{s}^2)$ and the von Mises stress $\sigma_\mathrm{e} = \sqrt{3J_2}$.

   (d) If we assume that the stress components which were impossible to determine varies in the range $(-\sigma_0, \sigma_0)$, determine the range of the von Mises stress.



2. Determine the dimension of the height h of the two span beam shown below, such that the von Mises stress $\sigma_\mathrm{e} = \sqrt{3J_2}$ is everywhere smaller than 355 MPa (steel S 355) and it should sustain a vertical load of 100 kN in an arbitrary position. The profile of the cross section is I, having dimensions of $h = 2b$, $t_\mathrm{f} = \frac{3}{2}t_\mathrm{w}$ and $t_\mathrm{w} = \frac{1}{50}h$. The spans are $L = 6$ m. The middle support width in the $b/2$ (at the ends half of it) and the width is the width of the flange. You can analyse the beam as an idealized I-profile, such that the bending moment M is completely carried by the flanges and the web takes all the shear force. In addition you can assume that the normal stresses are constant in the thickness direction of the flanges.

   Where is the most dangerous place of the load?



3. Is the following stress field in equilibrium if there are no body forces

$$\sigma_x = \frac{\sigma_0}{L^2}\left(3x^2 + 4xy - 8y^2\right), \qquad \sigma_y = \frac{\sigma_0}{L^2}\left(2x^2 - xy + 3y^2\right),$$
$$\tau_{xy} = \frac{\sigma_0}{L^2}\left(\tfrac{1}{2}x^2 - 6xy - 2y^2\right), \qquad \sigma_z = \tau_{yz} = \tau_{zx} = 0,$$

where σ_0 and L are constants.

4. In a rectangular, rectilinear (x, y, z)-coordinate system the stress matrix is given as

$$\sigma = \begin{bmatrix} (1-\xi^2)\eta + \frac{2}{3}\eta^2 & -(4-\eta^2)\xi & 0 \\ -(4-\eta^2)\xi & -\frac{1}{3}(\eta^3 + 12\eta) & 0 \\ 0 & 0 & (3-\xi^2)\eta \end{bmatrix} \sigma_0,$$

where $\xi = x/L, \eta = y/L$, σ_0 and L are constants.

(a) Is the stress state in equilibrium in the absense of body forces?

(b) Determine traction vector at $(2L, -L, 6L)$, on the plane $3x + 6y + 2z = 12L$.

5. A plate with linearly varying depth and having a constant thickness t is loaded by a shear traction at its free end having the resultant F. Determine the boundary conditions of the problem at every part of the boundary. Derive the distribution of the shear stress $\tau_{xy}(x, y)$ assuming that the normal stress σ_x follows from the elementary beam theory.