FEM advanced course

8. exercise – 3D solid element

Home assignment 8. Code a linear isoparametric trilinear (8 node) continuum element. The element stiffness matrix is computed as

$$\boldsymbol{K}^{(e)} = \int_{\Omega^{(e)}} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{C} \boldsymbol{B} \, \mathrm{d}V,$$

where $\boldsymbol{B}$ is the strain-displacement matrix relating virtual strains to virtual dsplacements $\delta\boldsymbol{\varepsilon} = \boldsymbol{B}\delta\boldsymbol{q}^{(e)}$. The B-matrix can be partitioned in the nodal contributions as

$$\boldsymbol{B} = [\boldsymbol{B}_1, \boldsymbol{B}_2, \ldots, \boldsymbol{B}_8],$$

where the part related to node k is

$$\boldsymbol{B}_k = \begin{pmatrix} N_{k,X} & 0 & 0 \\ 0 & N_{k,Y} & 0 \\ 0 & 0 & N_{k,Z} \\ N_{k,Y} & N_{k,X} & 0 \\ 0 & N_{k,Z} & N_{k,Y} \\ N_{k,Z} & 0 & N_{k,X} \end{pmatrix}$$

Use $2 \times 2 \times 2$ Gauss-Legendre integration. The St. Venant-Kirchhoff material stiffness matrix $\boldsymbol{C}$ can be found from Wriggers' book: equation (3.273). You can have a look of the geometry Jacobian matrix and interpolation functions in my lecture notes, page 252, section 12.2.4 https://webpages.tuni.fi/rakmek/mei_55200/pruju/knrm.pdf. Misprint in equation (12.32), it should be

$$N_i(\xi, \eta, \zeta) = \tfrac{1}{8}(1 + \xi_i \xi)(1 + \eta_i \eta)(1 + \zeta_i \zeta).$$

As a startig point compute the volume of an element $\int_{\Omega^{(e)}} \mathrm{d}V$.

Analysis cases

Analyse the following cases.

1. Compute the bar in tension. Use such boundary conditions that the transverse deformations can take place freely. Use Young's modulus $E = 0.1$ GPa and Poisson's ratio $\nu = 0.3, 0.45, 0.49$ and 0.49999. The tensile load acting on the free 4 kN. Length of the bar is 100 mm and the area of the cross-section is 100 mm^2. Use square cross-section.

2. Analyse the bar in pure bending. Use bending moment 20 Nm about the y-axis..

3. Analyse the tension case if all displacements are supressed at the support plane. Preferebly use more than one element to see the deformations.