

FEM advanced course

7. exercise – path following algorithm

1. Use continuation method to solve the equilibrium path of the symmetrically loaded von Mises truss beyond the limit point. The equilibrium equation is then

$$f(q, \lambda) = (\sin^2 \alpha)q - \frac{3}{2}(\sin \alpha)q^2 + \frac{1}{2}q^3 - \lambda = 0,$$

where $q = v/L$ and $\lambda = P/EA$. You can use for the angle α the value $\alpha = 30^\circ$. Use the spherical constraint equation

$$c(q, \lambda) = \Delta q^2 + \psi^2 \Delta \lambda^2 - \Delta s^2 = (q - q_0)^2 + \psi^2 (\lambda - \lambda_0)^2 - \Delta s^2 = 0,$$

where q_0, λ_0 are the values of the last converged step and the block elimination strategy.

Use $\alpha = 30^\circ$ and initial step $\Delta \lambda_0 = 0.02$. Carry out calculations for two steps with convergence tolerances $\epsilon_{\text{rel}} = 5 \cdot 10^{-2}$ and $\epsilon_{\text{abs}} = 10^{-3}$.

Home assignment 7. Code the path following (continuation) algorithm using consistent linearization and spherical type constraint equation

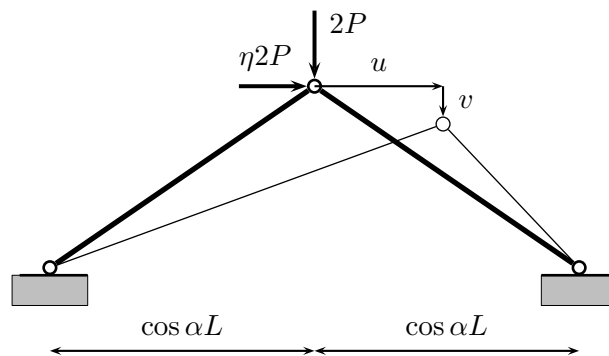
$$c(\mathbf{q}, \lambda) = \Delta \mathbf{q}^T \Delta \mathbf{q} + \psi^2 \Delta \lambda^2 - \Delta s^2 = 0,$$

where the scaling parameter ψ can be set at the first load step to give equal balance between the displacement and load terms.

Use your total Lagrangian truss element in the evaluation of internal forces and stiffness (Jacobian of the unbalanced force) matrix.

Analysis cases

Compute the complete equilibrium path of the von Mises truss.



Analyse the following cases.

1. Use $\alpha = 30^\circ$ and symmetric loading, i.e. $\eta = 0$. Stop traversing the path when $\lambda_{\text{max}} = 0.03$ ($\lambda = P/EA$). Test computation with different initial load steps. Start with $\Delta \lambda_0 = 0.002$.

2. Use $\alpha = 60^\circ$ and both symmetric $\eta = 0$ and unsymmetric loading $\eta = 0.1$. Stop traversing the path when $\lambda_{\max} = 0.2$ ($\lambda = P/EA$). Test computation with different initial load steps. Start with $\Delta\lambda_0 = 0.01$.

Plot the equilibrium paths in dimensionless axis $q_1 = u/L, q_2 = v/L$ and λ and report convergence tolerances, number of steps and iterations needed etc.

If you have not succeeded to implement the geometrically non-linear truss element you can use the equilibrium equations given in the lecture notes and in Home assignment 1.

Code structure for the continuation algorithm, block elimination strategy

- For all load increments.
 1. Predictor phase
 - (a) starting point $(\mathbf{q}_0, \lambda_0)$ such that $\mathbf{f}(\mathbf{q}_0, \lambda_0) = \mathbf{0}$.
 - (b) choose steplength Δs
 - (c) evaluate $\mathbf{K}_0 = \mathbf{f}'(\mathbf{q}_0, \lambda_0) = \partial\mathbf{f}/\partial\mathbf{q}|_0$
 - (d) factorize $\mathbf{K}_0 = \mathbf{LDU}$
 - (e) solve $\mathbf{K}_0\mathbf{q}_p = \mathbf{p}_r$
 - (f) compute $\Delta\lambda_0$ from the constraint equation c
 - (g) select the direction for traversing $\delta\lambda_0 = \pm\Delta\lambda_0$
 - (h) update $\mathbf{q}_1 = \mathbf{q}_0 + \delta\lambda_0\mathbf{q}_p$ and $\lambda_1 = \lambda_0 + \delta\lambda_0$
 2. Corrector phase: iterate $i = 1, \dots$,
 - (a) evaluate $\mathbf{f}_i = \mathbf{f}(\mathbf{q}_i, \lambda_i)$
 - (b) decide if the Jacobian should be updated
if yes, then compute $\mathbf{K}_i = \mathbf{f}'(\mathbf{q}_i, \lambda_i)$ and factorize \mathbf{K}_i
 - (c) solve $\mathbf{K}_i\delta\mathbf{q}_f = -\mathbf{f}_i$ and $\mathbf{K}_i\mathbf{q}_p = \mathbf{p}_r$
 - (d) compute $\delta\lambda_i = -(c + \mathbf{b}^T\delta\mathbf{q}_f)/(e + \mathbf{b}^T\mathbf{q}_p)$ and $\delta\mathbf{q}_i = \delta\mathbf{q}_f + \delta\lambda_i\mathbf{q}_p$
 - (e) update $\mathbf{q}_{i+1} = \mathbf{q}_i + \delta\mathbf{q}_i$ and $\lambda_{i+1} = \lambda_i + \delta\lambda_i$
 - (f) if convergence set $\mathbf{q}_0 = \mathbf{q}_{i+1}$ and $\lambda_0 = \lambda_{i+1}$ and go to a new predictor step 1.

Here $\mathbf{b} = \partial c/\partial\mathbf{q}$ and $e = \partial c/\partial\lambda$.

Solution report should be returned in Moodle at latest Sunday, March 27