FEM advanced course

5. exercise – non-linear truss element

Home assignment 5. In the lectures the total Lagrangian two node 1,2 or 3-dimensional truss element formulation was presented and the pseudocode for load incrementing and iteration at each load increment is the following.

1. Load steps $n = 1, 2, \ldots, n_{\max}$. Increment load $\boldsymbol{p}_n = \boldsymbol{p}_{n-1} + \Delta \boldsymbol{p}_n$ and set $\boldsymbol{q}_n^{(0)} = \boldsymbol{q}_{n-1}$
   (a) Iterate $i = 0, 1, 2, \ldots, i_{\max}$
      i. In each element extract $\boldsymbol{u}$ from $\boldsymbol{q}$ and compute $\boldsymbol{x} = \boldsymbol{X} + \boldsymbol{u}$ and strains
      $$\varepsilon_n^{(i)} = \frac{1}{\ell_0^2} \tfrac{1}{2} (\tilde{\boldsymbol{X}} + \tilde{\boldsymbol{x}})^{\mathrm{T}} \begin{pmatrix} \boldsymbol{I} & -\boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{I} \end{pmatrix} \tilde{\boldsymbol{u}}_n^{(i)}$$
      ii. Compute internal force vector from element contributions
      $$\tilde{\boldsymbol{r}} = \frac{EA_0}{\ell_0} \varepsilon_n^{(i)} \begin{pmatrix} \boldsymbol{I} & -\boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{I} \end{pmatrix} \tilde{\boldsymbol{x}}_n^{(i)} = \frac{N_n^{(i)}}{\ell_0} \begin{pmatrix} \boldsymbol{I} & -\boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{I} \end{pmatrix} \tilde{\boldsymbol{x}}_n^{(i)}$$
      iii. Assemble the global stiffness matrix $\boldsymbol{K}_n^{(i)} = \boldsymbol{K}_0(\boldsymbol{X}) + \boldsymbol{K}_u(\boldsymbol{X}, \boldsymbol{u}_n^{(i)}) + \boldsymbol{K}_\sigma(\varepsilon_n^{(i)})$,
      iv. Compute the global residual force $\boldsymbol{f}_n^{(i)} = \boldsymbol{r}_n^{(i)} - \boldsymbol{p}_n$
      v. Solve the linearized system $\boldsymbol{K}_n^{(i)} \delta \boldsymbol{q}_n^{(i)} = \boldsymbol{f}_n^{(i)}$, *notice: δ symbol here means the iterative change!*
      vi. Update global displacement vector $\boldsymbol{q}_n^{i+1} = \boldsymbol{q}_n^{(i)} - \delta \boldsymbol{q}_n^{(i)}$

The node coordinates are denoted as $\boldsymbol{X}_A$ and $\boldsymbol{X}_B$ in the initial configuration and $\boldsymbol{x}_A = \boldsymbol{X}_A + \boldsymbol{u}_A$ and $\boldsymbol{x}_B = \boldsymbol{X}_B + \boldsymbol{u}_B$ in the deformed configuration. Furthermore $\boldsymbol{X} = \boldsymbol{X}_B - \boldsymbol{X}_A$, $\boldsymbol{x} = \boldsymbol{x}_B - \boldsymbol{x}_A$, $\boldsymbol{u} = \boldsymbol{u}_B - \boldsymbol{u}_A$ etc., and

$$\tilde{\boldsymbol{X}} = \begin{pmatrix} \boldsymbol{X}_A \\ \boldsymbol{X}_B \end{pmatrix}, \quad \text{etc.}$$

You can use the MiniTruss Matlab-code by Prof. emer. Steen Krenk and extend it to non-linear problems, if you want.

The normal force defined as

$$N = EA_0 \varepsilon,$$

where ε is the Green-Lagrange strain, N is the PK2-type stress resultant. Derive expression for Cauchy type stress resultant $N_\sigma = \int_A \sigma \mathrm{d}A$. If the material is assumed to be incompressible, derive the expression also for the Cauchy stress σ.

Analysis case

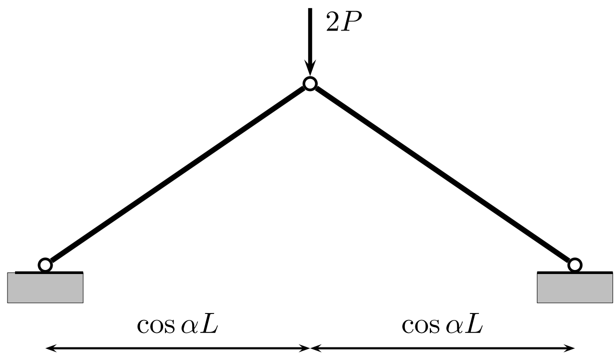
1. With your element analyse the bar problem used in the previous assignments loaded only by the horizontal force on the unconstrained end. Compare the results.

2. Compute the symmetric equilibrium path of the von Mises truss from the first home assignment. Draw the equilibrium path and monitor the convergence.

   Length and the initial angle of the bars at the initial state are L and α, respectively, and the axial stiffness equals to EA_0. Non-dimensional load parameter λ is $\lambda = P/EA_0$. The bars are assumed to be absolutely rigid in bending. See, lecture notes *Computational Techniques for the non-linear analysis of structures*, Example 1.3.1.

   Select the angle $\alpha = 30°$ and compute the equilibrium path up to $\lambda_{\max} = 0.19 \sin^3 \alpha$ using

   (a) a small load step, e.g $\Delta\lambda = \lambda_{\max}/100$ and
   (b) a large load step, e.g. $\Delta\lambda = \lambda_{\max}/2$.



Write a short report on the computations and on the program you have made.

Solution report should be returned in Moodle before exercise 7.