FEM advanced course

3. exercise – kinematics, time rates, constitutive model

1. A one-dimensional motion of a bar is given as $x = (1 + t/t_0)X,\ x \in [0, 2L]$.
   (a) Determine the material time derivative of the Green-Lagrange strain $\boldsymbol{E}$.
   (b) Determine the material time derivative of the Almansi strain $\boldsymbol{e} = \frac{1}{2}(\boldsymbol{I} - \boldsymbol{F}^{-T}\boldsymbol{F}^{-1})$.
   (c) Determine the rate of deformation tensor $\boldsymbol{d}$.
   (d) For this particular motion, show that the following equations are correct: $\boldsymbol{d} = \boldsymbol{F}^{-\mathrm{T}}\dot{\boldsymbol{E}}\boldsymbol{F}^{-1}$ and $\boldsymbol{d} = \dot{\boldsymbol{e}} + \boldsymbol{l}^{\mathrm{T}}\boldsymbol{e} + \boldsymbol{e}\boldsymbol{l}$, where $\boldsymbol{l}$ is the spatial velocity gradient.
   (e) Temperature field in the bar is given as $T = T_0(X/L)(t/t_0)^2$. Determine the spatial form $T(x,t)$? Using $T(x,t)$, determine $\dot{T}$ and show that it equals to $\mathrm{d}T(X,t)/\mathrm{d}t$.

2. Extend your non-linear 1D-truss element code to handle incompressible neo-Hookean material model having the strain energy function

$$W(I_C) = \frac{1}{2}\mu(I_C - 3), \quad \text{and} \quad \boldsymbol{S} = 2\frac{\partial W}{\partial \boldsymbol{C}} \tag{1}$$

   where μ is the shear modulus, $I_C = \mathrm{tr}\,\boldsymbol{C}$ and $\boldsymbol{S}$ is the second Piola-Kirchhoff stress. Solve the problem with boundary conditions $u(0) = 0$ and force H is acting at $X = L_0$. Plot also the Cauchy stress-displacement curve. You can use same values as in home assignment 2, i.e. $L_0 = 100$ mm, $A_0 = 10$ mm^2, and the shear modulus $\mu = 33.333$ MPa. Compute to the maximum load 6 kN in tension and same in compression. Compare results with the St. Venant constitutive model.

Home assignment 3. Extend your non-linear 1D-truss element code to handle incompressible Mooney-Rivlin material model having the strain energy function (3.115) in the study book

$$W(\lambda_1, \lambda_2, \lambda_3) = \frac{1}{2}\mu_1(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) - \frac{1}{2}\mu_2(\lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2} - 3). \tag{2}$$

The principal PK2-stresses can now be obtained as

$$S_i = \frac{1}{\lambda_i}\frac{\partial W}{\partial \lambda_i}. \tag{3}$$

1. Solve the problem with boundary conditions $u(0) = 0$ and force H is acting at $X = L_0$. Use one element.

2. Solve the problem of a hanging rubber band. Displacement is supressed at $X = 0$ and the other end at $X = L_0$ is free. The loading is now the gravity load $\rho_0 g$ in the positive X-axis direction. Use 1, 2 and 100 elements.

Choose μ_1 and μ_2 such that the initial response is the same as in before. For the density use 1000 times the desity of rubber, i.e. about $\rho_0 = 1.1 \cdot 10^6$ kg/m^3.

You can use same dimensions for initial length and cross-section area as in problem 2. Plot also the Cauchy stress-displacement curve.

Solution report should be returned in Moodle prior to exercise 5