Application of ADI Splitting Methods to Two-Dimensional Building Envelope System Solvers

Anne Paepcke, Andreas Nicolai, John Grunewald
Institute of Building Climatology, Dresden Technical University, Germany

Tampere, 2/06/2011
Motivation

Heat Transfer Equation

Thermal Balance

\[ 0 = c_T \rho \frac{\partial T}{\partial t} + \frac{\partial}{\partial x_k} q_{\text{cond},k} \]

\[ q_{\text{cond},k} = -\lambda \frac{\partial T}{\partial x_k} \]

Numerical Discretisation

- Space: Two-dimensional grid
- Time: Implicit or explicit time stepping method
ADI-Method

**ADI Peaceman-Rachefort**

\[
\begin{align*}
\frac{c_T \rho}{\Delta t} T^{n+\frac{1}{2}} - T^n &= -\frac{\partial}{\partial x} \left(-\lambda \frac{\partial T}{\partial x}\right)^{n+\frac{1}{2}} - \frac{\partial}{\partial y} \left(-\lambda \frac{\partial T}{\partial y}\right)^n \\
\frac{c_T \rho}{\Delta t} T^{n+1} - T^{n+\frac{1}{2}} &= -\frac{\partial}{\partial x} \left(-\lambda \frac{\partial T}{\partial x}\right)^{n+\frac{1}{2}} - \frac{\partial}{\partial y} \left(-\lambda \frac{\partial T}{\partial y}\right)^{n+1}
\end{align*}
\]

- Implicitly treated
- Explicitly treated
ADI-Method

**ADI Peaceman-Rachefort**

\[ c_T \rho \frac{T^{n+\frac{1}{2}} - T^n}{\Delta t} = -\frac{\partial}{\partial x} \left(-\lambda \frac{\partial T^{n+\frac{1}{2}}}{\partial x} \right) \]

\[ -\frac{\partial}{\partial y} \left(-\lambda \frac{\partial T^n}{\partial y} \right) \]

\[ c_T \rho \frac{T^{n+1} - T^{n+\frac{1}{2}}}{\Delta t} = -\frac{\partial}{\partial x} \left(-\lambda \frac{\partial T^{n+1}}{\partial x} \right) \]

\[ -\frac{\partial}{\partial y} \left(-\lambda \frac{\partial T^{n+1}}{\partial y} \right) \]

Implicitly treated
 Explicitly treated
ADI-Method

**ADI Peaceman-Rachefort**

\[
\begin{align*}
c_T \rho \frac{T^{n+\frac{1}{2}} - T^n}{\Delta t} &= -\frac{\partial}{\partial x} \left( -\lambda \frac{\partial T}{\partial x} \right)^{n+\frac{1}{2}} - \frac{\partial}{\partial y} \left( -\lambda \frac{\partial T}{\partial y} \right)^n \\
c_T \rho \frac{T^{n+1} - T^{n+\frac{1}{2}}}{\Delta t} &= -\frac{\partial}{\partial x} \left( -\lambda \frac{\partial T}{\partial x} \right)^{n+\frac{1}{2}} - \frac{\partial}{\partial y} \left( -\lambda \frac{\partial T}{\partial y} \right)^{n+1}
\end{align*}
\]

Implicitly treated
Explicitly treated
ADI-Method

Properties of classical ADI

- Implicit solution of a set of one-dimensional equations
- Tridiagonal Jacobian matrix
- Easy implementation in the case of regular grids
- Unconditional stable when applied to parabolic equations
- Loss of accuracy in presence of mixed-term-derivatives
- Limited suitability to parallelisation algorithms
ADI-Method

Properties of classical ADI

- Implicit solution of a set of one-dimensional equations
- Tridiagonal Jacobian matrix
- Easy implementation in the case of regular grids
- Unconditional stable when applied to parabolic equations
- Loss of accuracy in presence of mixed-term-derivatives
- Limited suitability to parallelisation algorithms
Numerical Tests

Aluminum Insulation Concrete

Application of ADI Splitting Methods to Two-Dimensional Building Envelope System Solvers
Numerical Tests

Results

- Test for accuracy: error estimated by comparison to implicit method
- Estimation of adaptive time step sizes
- Strict step size limitation for all cases
- Strong performance decrease with increasing geometrical complexity

Average time step size [s]: 0-10h/10h-2d

<table>
<thead>
<tr>
<th></th>
<th>Sequential</th>
<th>Classical</th>
<th>Additive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>26/27</td>
<td>175/252</td>
<td>32/95</td>
</tr>
<tr>
<td>Case II</td>
<td>4/5</td>
<td>86/189</td>
<td>29/79</td>
</tr>
<tr>
<td>Case III</td>
<td>0.1/0.1</td>
<td>37/75</td>
<td>9/23</td>
</tr>
</tbody>
</table>
Occurrence of Mixed-Term-Derivatives

**Heat flux inside material \( G_i \):**

\[
q_{\text{cond},k\mid G_i} = -\lambda_i(T) \frac{\partial T}{\partial x_k}
\]

**Helmholtz decomposition:**

\[
q_{\text{cond}} = \nabla \Phi + \nabla \times \Psi
\]

**Directional splitting:**

\[
\frac{\partial}{\partial x} q_{\text{cond},1} = \frac{\partial^2}{\partial x^2} \Phi + \frac{\partial^2}{\partial x \partial y} \Psi
\]

\[
\frac{\partial}{\partial y} q_{\text{cond},2} = \frac{\partial^2}{\partial y^2} \Phi - \frac{\partial^2}{\partial x \partial y} \Psi
\]
Occurrence of Mixed-Term-Derivatives

**Heat flux inside material** $G_i$:

$$q_{\text{cond},k|G_i} = -\lambda_i(T) \frac{\partial T}{\partial x_k}$$

**Helmholtz decomposition:**

$$q_{\text{cond}} = \nabla \Phi + \nabla \times \Psi$$

**Directional splitting:**

$$\frac{\partial}{\partial x} q_{\text{cond},1} = \frac{\partial^2}{\partial x^2} \Phi + \frac{\partial^2}{\partial x \partial y} \Psi$$

$$\frac{\partial}{\partial y} q_{\text{cond},2} = \frac{\partial^2}{\partial y^2} \Phi - \frac{\partial^2}{\partial x \partial y} \Psi$$
Occurrence of Mixed-Term-Derivatives

Aluminum Insulation Concrete

Temperature [°C]

18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Tampere, 2/06/2011

Application of ADI Splitting Methods to Two-Dimensional Building Envelope System Solvers
Conclusions

**ADI-methods in BES simulation**
- Acceptable accuracy requires small time step size
- ADI method is unsuitable for problems with discontinuous material properties, all balance types

**Alternatives to direct application of ADI**
- ADI achieves not an exact but a good approximate initial solution
- ADI can be easily transformed into a matrix preconditioner
- Alternative approach: Use of splitting preconditioning strategies combined with iterative linear equation system solvers
Thank you for your attention!
References


