Self-Backhauling Full-Duplex Access Node with Massive Antenna Arrays:
Power Allocation and Achievable Sum-Rate

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Abstract—This paper analyzes a self-backhauling inband full-duplex access node that has massive antenna arrays for transmission and reception. In particular, the optimal transmit powers for such a system are solved in a closed form, taking into account the self-interference as well as backhaul capacity requirements and incorporating the role of downlink–uplink traffic ratio in sum-rate maximization. Numerical results are also provided, where the obtained analytical expressions are evaluated with realistic system parameter values. All in all, the presented theory and the numerical results provide insights into the proposed system, indicating that a self-backhauling access node could greatly benefit from being capable of inband full-duplex communication.

I. INTRODUCTION

Recently, several works have demonstrated the feasibility of inband full-duplex communications, where the transmission and reception are done simultaneously using the same center-frequency [1], [2]. This is made possible by the various advanced techniques for cancelling the own transmit signal in the receiver, which is necessary in order to observe the received signal of interest. The greatest benefit of inband full-duplex communications is obviously the doubling of the spectral efficiency, which stems from the fact that the whole bandwidth can be used for both transmission and reception.

The inband full-duplex capability opens also other possibilities beyond the mere increase in the data rate of a system. In particular, combining it with the principle of self-backhauling has been of recent interest to the research community [3], [4]. What self-backhauling means is that an access node (AN) is wirelessly handling its own backhaul connection, such that no cabling or dedicated microwave links are required. Furthermore, if self-backhauling is done in full-duplex mode, no additional spectral or temporal resources are needed, making the backhaul connection entirely transparent [5]. This is especially beneficial for densely populated cells, where it is infeasible to provide a high-speed wired backhaul connection for each AN. For more information regarding possible backhaul solutions in 5G networks, see [6].

In this work, a self-backhauling full-duplex AN with a massive antenna array is studied and analyzed. The large array allows the AN to do very efficient beamforming and thereby avoid multi-user interference [4], as well as partially null its own self-interference (SI) [7], [8]. In addition to adapting zero-forcing beamforming procedures to the system at hand, the main contributions of this article are as follows:

- providing closed-form solutions for the optimal transmit powers of the AN and the user equipments (UEs), taking into account the SI and the required backhauling capacity;
- deriving the respective sum-rate expressions for the considered cell, including both uplink and downlink traffic.

Overall, these results provide information regarding the feasibility of the considered self-backhauling full-duplex AN, especially in terms of the necessary transmit power allocation which has not been addressed in this context before.

II. SYSTEM MODEL

Let us consider a system illustrated in Fig. 1, where a full-duplex AN with separate transmit and receive antenna arrays is serving half-duplex UEs simultaneously in the uplink and in the downlink, while also backhauling itself with a full-duplex capable backhaul node (BN). All of this is done using the same center-frequency, which is made possible by beamforming and digital SI cancellation. Together with the passive isolation provided by the physical separation between the transmit and receive antenna arrays, these techniques suppress the SI signal and also ensure that the signals of interest can be spatially separated. In order to obtain the relevant signal-to-interference-plus-noise ratios (SINRs) for the considered system, and consequently the rate expressions, let us first define the overall signal received by the UEs and the BN. Denoting the combined amount of downlink UEs and spatial streams transmitted to the BN by \(M_t\), and the total number of transmit antennas at the AN by \(N_t\) such that \(N_t >> M_t\), it can be written as follows:

\[ y_t = H_t x + z, \] (1)
where $H_t \in \mathbb{C}^{M_t \times N_t}$ is the channel matrix between the AN and the intended receivers, $x \in \mathbb{C}^{N_t \times 1}$ is the transmit signal of the AN and $z \in \mathbb{C}^{M_t \times 1}$ represents the different noise and interference sources. In this paper, Rayleigh fading between all communicating parties is assumed, which means that $H_t \sim \mathcal{CN}(0, L)$, where $L = \text{diag}(L_1, L_2, \ldots, L_M)$ is a diagonal matrix containing the path loss normalized fading variances to the different receivers.

The transmit signal $x$ is formed from the actual transmit data as follows:

$$x = W \Pi q,$$

(2)

where $W \in \mathbb{C}^{N_t \times M_t}$ is a zero-forcing (ZF) precoding matrix, $\Pi \in \mathbb{C}^{M_t \times M_t}$ is a diagonal matrix containing the square roots of the transmit powers $p_k$ allocated for each symbol in its diagonal, and $q \in \mathbb{C}^{M_t \times 1}$ contains the normalized transmit data symbols. The ZF precoding matrix $W$ takes care of minimizing the amount of SI radiated to the receive antennas in the AN, while maximizing the amount of signal power directed towards the intended recipients. Denoting the SI channel matrix between the transmit and receive antennas by $H_i \in \mathbb{C}^{N_i \times N_t}$, where $N_r < N_t$ is the number of receive antennas at the AN, and assuming that the AN has full channel state information (CSI) available, the ZF precoding matrix for the downlink transmission can be written as [8]

$$W = H_i^H (H_i H_i^H)^{-1} \Lambda,$$

(3)

where $H^H = [H_t^H \ H_i^H]$, $(\cdot)^H$ denotes the Hermitian transpose, and $\Lambda \in \mathbb{C}^{M_t + N_t \times M_t}$ is a non-square diagonal normalization matrix containing the individual normalization factors $\lambda_k$. The purpose of the normalization matrix is to ensure that precoding does not affect the effective powers of the data symbols. The diagonal elements of the normalization matrix can be shown to be as follows [8]:

$$\lambda_k = \sqrt{\lambda_k} = \sqrt{L_k (N_t - M_t - N_r)}.$$

Now, taking into account the ZF precoding, all the received signals can be rewritten as follows:

$$y_t = H_t x + z = H_t W \Pi q + z = \tilde{\Lambda} \Pi q + z,$$

(4)

where $\tilde{\Lambda} \in \mathbb{C}^{M_t \times M_t}$ refers to $\Lambda$ with the zero rows removed. Component wise, this result can be written as

$$y_{t,k} = \sqrt{\lambda_k} (N_t - M_t - N_r) p_k q_k + z_k,$$

(5)

where $k = 1, 2, \ldots, M_t$ and $\sqrt{\lambda_k}$ is the $k$th diagonal element of $\Pi$.

The SINR of the $k$th data signal, received either by an UE or the BN, can then be expressed as follows [8]:

$$\text{SINR}_{t,k} = \frac{\mathbb{E}[|y_{t,k} - z_k|^2]}{\mathbb{E}[|z_k|^2]} = \frac{L_k (N_t - M_t - N_r) p_k}{\sigma^2_{t,N} + \sigma^2_{t,I,k}},$$

(6)

where $\sigma^2_{t,N} + \sigma^2_{t,I,k}$ is the variance of the noise-plus-interference term $z_k$, divided into the receiver noise and interference components.

Using nearly an identical derivation as for the AN transmit signals (cf. [8]), the SINR for the $j$th data signal transmitted by the uplink UEs or the BN, and received by the AN, can be expressed as

$$\text{SINR}_{r,j} = \frac{L_j (N_r - M_r) p_j}{\sigma^2_{r,N} + \sigma^2_{r,I,j}},$$

(7)

where $L_j$ is the path loss normalized fading variance of the $j$th signal stream, $M_r < N_r$ is the number of received signal streams, $p_j$ is the corresponding transmit power, and $\sigma^2_{r,N} + \sigma^2_{r,I,j}$ is the variance of the noise-plus-interference term. Note that in this work it is assumed that the BN does not do any beamforming itself, meaning that all the processing is done by the AN. This assumption is made in order to make the analysis more straightforward.

Hence, using (6) and (7) and making the typical assumption of Gaussian distributed noise and interference, the total data rates for the downlink and uplink can be expressed as

$$R_d = \sum_{k \in DL} \log_2 (1 + \text{SINR}_{t,k}) = D \log_2 (1 + \Gamma_d P_d),$$

(8)

$$R_u = \sum_{j \in UL} \log_2 (1 + \text{SINR}_{r,j}) = U \log_2 (1 + \Gamma_u P_u),$$

(9)

where

$$\Gamma_d = \frac{L_{UE} (N_t - N_r - D - M_t^{BH})}{D \sigma^2_n + \alpha P_{AN}},$$

(10)

$$\Gamma_u = \frac{L_{UE} (N_r - U - M_r^{BH})}{\sigma^2_n + \alpha P_{AN}},$$

(11)

and $D$ is the number of UEs in the downlink, $U$ is the number of UEs in the uplink, $L_{UE}$ is the path loss normalized fading variance between the UEs and the AN, $M_t^{BH}$ and $M_r^{BH}$ are the numbers of transmitted and received backhaul data streams, $\sigma^2_n$ is the noise floor in all the receivers, $\alpha$ is the total amount of SI cancellation in the AN, $P_d$ is the amount of transmit power used for the downlink data streams, and $P_u$ is the transmit power of an individual UE. In order to simplify the analysis of the considered system, it is assumed that the AN is always transmitting with the same total transmit power, denoted by $P_{AN}$, and $P_d$ only determines how much of it is used for downlink transmission. In addition, it is also assumed that the path loss is the same for all the UEs, and that the uplink and downlink UEs are sufficiently separated such that there is no significant inter-user-interference between them.

The data rates for the backhaul connection can also be easily derived using (6) and (7), and they are written as follows:

$$R_{BH}^d = \sum_{j \in BH} \log_2 (1 + \text{SINR}_{r,j}) = M_r^{BH} \log_2 (1 + \Gamma_d^{BH} P_d^{BH}),$$

(12)

$$R_{BH}^u = \sum_{k \in BH} \log_2 (1 + \text{SINR}_{t,k}) = M_t^{BH} \log_2 (1 + \Gamma_u^{BH} (P_{AN} - P_d)),$$

(13)

where

$$\Gamma_d^{BH} = \frac{L_{BH} (N_r - U - M_r^{BH})}{M_r^{BH} \sigma^2_n + \alpha P_{AN}},$$

(14)
\[
\Gamma_u = \frac{L_{BH} (N_t - N_r - D - M^B_H)}{M^B_H \sigma_n^2}, \tag{15}
\]

\(L_{BH}\) is the path loss normalized fading variance between the AN and the BN, and \(P^B_H\) is the transmit power of the BN. In this work, it is assumed that the BN is capable of perfect SI cancellation due to it being a large infrastructure node. Note that now \(R^B_H\) is the achievable backhaul rate from BN to AN while \(R^B_u\) is the corresponding rate from AN to BN.

### III. SUM-RATE AND TRANSMIT POWER ANALYSIS

Our objective is to maximize the sum-rate of the uplink and downlink by adjusting the different transmit powers accordingly. Hence, the objective function can be defined as follows:

\[
S(P_d, P_u) = R_d + R_u \tag{16}
\]

\[
= D \log_2 (1 + \Gamma_d P_d) + U \log_2 (1 + \Gamma_u P_u).
\]

This optimization problem is subject to certain constraints, the most important of which is perhaps the requirement for backhaul capacity. In particular, the AN must be able to backhaul itself, resulting in the following conditions:

\[
g_1(P_d) = R_d - R^B_H \leq 0 \tag{17}
g_2(P_d, P_u) = R_u - R^B_u \leq 0. \tag{18}
\]

In addition to these constraints, both of the transmit powers also have strict upper limits, resulting in the inequalities below:

\[
g_3(P_u) = P_u - P_{UE} \leq 0 \tag{19}
g_4(P_d) = P_d - P_{AN} \leq 0, \tag{20}
\]

where \(P_{UE}\) is the maximum transmit power of each UE.

Finally, one equality constraint is added to the optimization problem in order to ensure a reasonable relationship between the magnitudes of the uplink and downlink data rates. The reason for this is that typically the required uplink data rate is only a fraction of the downlink data rate, and hence, when analyzing the sum-rate, this must also be taken into consideration [9]. In this work, a fixed ratio between the uplink and downlink data rates is used to ensure that the capacity within the cell is divided in a meaningful way. The resulting equality constraint can be written as follows:

\[
h_1(P_d, P_u) = \rho R_d - R_u = 0, \tag{21}
\]

where \(\rho\) is the desired ratio between the uplink and downlink data rates.

Next, in order to simplify the optimization problem further, the objective function is transformed by expressing it as a base 2 exponential. This removes the logarithm, which will make the problem more easily tractable. Furthermore, since \(2^x\) is a monotonic function of \(x\), the transformed optimization problem is still equivalent to the original problem. Hence, we can express the new transformed objective function as

\[
\tilde{S}(P_d, P_u) = 2^{S(P_d, P_u)} = (1 + \Gamma_d P_d)^D (1 + \Gamma_u P_u)^U. \tag{22}
\]

Another simplification can be made by using the equality constraint in (21), which allows us to rewrite all the constraints and the objective function in terms of one transmit power. This results in a one-dimensional optimization problem, which is more straightforward to analyze. Transforming (21) first in the same way as the objective function, we can rewrite it as

\[
\tilde{h}_1(P_d, P_u) = (1 + \Gamma_d P_d)^D - (1 + \Gamma_u P_u)^U = 0, \tag{23}
\]

Using (23), we can then obtain an expression for the uplink transmit power as

\[
P_u = \frac{(1 + \Gamma_d P_d)^D - 1}{\Gamma_u}, \tag{24}
\]

where \(\Gamma_d\) and \(\Gamma_u\) are as defined in (10) and (11).

Equation (24) shows that under the data rate ratio constraint, \(P_u\) is in fact directly proportional to \(P_d\). This allows us to express the optimization problem with a one-dimensional objective function. In particular, using (23), we get

\[
\tilde{S}(P_d, P_u) = (1 + \Gamma_d P_d)^{D(1 + \rho)}. \tag{25}
\]

Furthermore, since the objective function is clearly monotonically increasing with respect to the argument \(P_d\), the sum-rate can be maximized by maximizing \(P_d\) under the given constraints in (17)–(20). The corresponding optimal uplink transmit power can then be calculated with (24).

The first upper bound for the downlink transmit power is given by (17), which specifies the relationship between the downlink data rate and the corresponding backhaul capacity. Assuming a fixed transmit power for the BN, the first bound can be rewritten as follows, using (8) and (12):

\[
P_d \leq P_d^I = \frac{(1 + \Gamma^B_H P^B_H)^{\frac{\rho D}{\beta D_M^u}} - 1}{\Gamma_d}. \tag{26}
\]

The second upper bound for \(P_d\) is provided by the uplink backhaul constraint in (18). With the help of (9), (13), and (24), we get

\[
P_d \leq P_{AN} = \frac{(1 + \Gamma_d P_d)^{\frac{\rho D}{\beta D_M^u}} - 1}{\Gamma_d}. \tag{27}
\]

Clearly there is no closed-form solution for an inequality of this form. In order to simplify this problem, the expression \(\frac{\rho D}{\beta D_M^u}\) can be approximated by a first-order Taylor series around the point \(P_d = \beta\), where \(\beta\) is a tunable parameter. This allows us to obtain the following approximative closed-form solution for (27):

\[
P_d \leq P_{AN}^I \approx \Gamma_u P_{AN} - (1 + \Gamma_d \beta - 1) \Gamma_d \beta - 1, \tag{28}
\]

where \(\tilde{\beta} = \frac{\rho D}{\beta D_M^u}\). Furthermore, when considering that usually most of the capacity is used for downlink, it can be deduced that \(P_d\) is under most circumstances close to \(P_{AN}\). Hence, the accuracy of (28) around the point \(P_d = P_{AN}\) should be maximized, meaning that \(\beta = P_{AN}\). In the forthcoming numerical results, (28) with \(\beta = P_{AN}\) is thereby used to evaluate the upper bound corresponding to (18). There it can be observed that it provides a good approximation of the actual solution under realistic system parameter values.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of AN transmit/receive antennas (N_t/N_r)</td>
<td>200/100</td>
</tr>
<tr>
<td>Number of downlink/uplink UEs ((D/U))</td>
<td>10/10</td>
</tr>
<tr>
<td>Number of downlink/uplink backhaul streams (M_r^{BH}/M_t^{BH})</td>
<td>12/6</td>
</tr>
<tr>
<td>Receiver noise floor (\sigma_d^2)</td>
<td>-90 dBm</td>
</tr>
<tr>
<td>Transmit power of the AN (P_{AN})</td>
<td>30 dBm</td>
</tr>
<tr>
<td>Maximum transmit power of the UEs (P_{UE})</td>
<td>25 dBm</td>
</tr>
<tr>
<td>Transmit power of the BN (P_d^{BH})</td>
<td>40 dBm</td>
</tr>
<tr>
<td>Amount of SI cancellation in the AN (\alpha)</td>
<td>-100 dB</td>
</tr>
<tr>
<td>Path loss between the AN and the UEs (L_{UE})</td>
<td>-90 dB</td>
</tr>
<tr>
<td>Path loss of the backhaul link (L_{BH})</td>
<td>-80 dB</td>
</tr>
<tr>
<td>Ratio between uplink and downlink data rates (\rho)</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The third bound restricting the maximum UE transmit power can be easily transformed into an upper bound for \(P_d\). Using (24), we can rewrite (19) as follows:

\[
P_d \leq P_d^{III} = \frac{(1 + \Gamma_u P_{UE})^{\rho_D} - 1}{\Gamma_u}. \tag{29}
\]

The final bound for \(P_d\) is given directly by (20), which simply states that

\[
P_d \leq P_d^{IV} = P_{AN}. \tag{30}
\]

Since the sum-rate is maximized by maximizing the downlink transmit power \(P_d\) under the given constraints, the solution to the optimization problem is simply

\[
P_d^* = \min \{ P_d^{I}, P_d^{II}, P_d^{III}, P_d^{IV} \}, \quad P_u^* = \frac{(1 + \Gamma_d P_u^{eD})^{\rho_D} - 1}{\Gamma_u}. \tag{31}
\]

The hereby obtained transmit powers are then used in (16) to calculate the corresponding maximum sum-rate. Even though this is strictly speaking a sub-optimal solution due to the approximations made when deriving \(P_d^{III}\), the numerical results show that the difference to the actual optimal solution is negligible.

### IV. Numerical Results

In order to obtain some further insights into the self-backhauling full-duplex AN, its maximum sum-rate under different circumstances is next evaluated numerically, using the equations derived in Section III. Table I lists the essential default parameters, which are used in evaluating the expressions unless otherwise mentioned.

In addition, to confirm the validity of the derived analytical results, the corresponding sum-rates obtained with a numeric optimization tool are also provided. In particular, the original optimization problem defined by (16)–(21) is given to the function \texttt{fmincon} in Matlab, and the corresponding results are then plotted together with the analytical result given by (31). This allows for an easy comparison to conclude that the two sets of results are very similar.

Figure 2(a) shows the maximum sum-rates with respect to the amount of SI cancellation, where the curves have been plotted for different values of \(M_r^{BH}\). The corresponding uplink and downlink transmit powers are shown in Fig. 2(b). As can be expected, a higher sum-rate is achieved when there are more spatial streams in the backhaul for transferring downlink data. However, the difference between \(M_r^{BH} = 18\) and \(M_r^{BH} = 24\) is already very small, indicating that with these values there are already other factors bottlenecks the performance. An important observation is also that, for all the considered parameter values, the analytical results match well with the numerically obtained results. This indicates that the approximations made in deriving the analytical equations do not compromise the accuracy of the numerical results.

When investigating which of the boundaries is limiting the downlink transmit power, Fig. 2(b) shows that with a low amount of SI cancellation, the UEs are using their maximum allowed transmit power, which is thereby the limiting factor. This is caused by the low SINR in the AN, which calls for a high transmit power from the UEs. When the amount of SI cancellation increases, the UEs start decreasing their transmit power to maintain the proper rate ratio. At this point, if the number of spatial streams in the backhaul link is small, the next limiting factor is the backhaul capacity for the downlink, represented by \(P_d^{IV}\). This is also caused by the limited SINR in the AN receiver due to the insufficient SI cancellation.
capabilities. However, this does not apply to the case with $M_r^{BH} = 24$, since there the high number of spatial streams ensures enough capacity in the downlink backhaul under all circumstances. In the case with $M_r^{BH} = 6$, on the other hand, the capacity of the downlink backhaul remains the limiting factor even with SI cancellation capabilities beyond 120 dB.

When the amount of SI cancellation goes beyond a certain value, the only limitation for the downlink transmit power is the uplink backhauling capacity, represented by $P_d^{BH}$. This ensures that a portion of the AN transmit power is used to backhaul the uplink data, thereby fulfilling the rate requirements. In Fig. 2(a), the effect of this constraint is seen as the saturation of the sum-rates, when the proportion of the transmit power used for the downlink cannot be increased anymore.

In Figs. 3(a) and 3(b), the sum-rates and optimal transmit powers are shown with respect to the ratio between the uplink and downlink data rates ($\rho$). The curves are plotted with three different values for the path loss normalized fading variance between the AN and the UEs ($L_{LUE}$). We see that now there is a certain value for $\rho$ that maximizes the sum-rate for a given path loss value. Looking at the transmit powers in Fig. 3(b), it can be seen that the sum-rate is maximized when the uplink power reaches its maximum value. In such a case, the value for $\rho$ allows for utilizing all the available resources to the fullest extent, and operating with any other data rate ratio will not take the full advantage of the transmit power boundaries.

Figure 3(a) also indicates that the optimal data rate ratio is dependent on the amount of path loss between the AN and the UEs. The reason for this is that the path loss affects the required uplink transmit power. With a lower path loss between the AN and the UEs, a lower uplink transmit power will result in the same SINR. This, on the other hand, results in the highest uplink transmit power being reached with a higher value for $\rho$, as can be seen in Fig. 3(b). Hence, the highest sum-rate is also achieved with a larger $\rho$. However, rate ratios of this magnitude are somewhat unrealistic when considering a practical system, and thereby the true achievable rate is bound to be less than the one obtained with optimal $\rho$ [9].

V. CONCLUSION

In this work, a self-backhauling full-duplex access node was studied and analyzed. In particular, closed-form solutions for the rate-maximizing transmit powers were derived, which took into account the effect of self-interference and the backhauling rate requirements. The highest achievable sum-rates were also numerically evaluated, providing insights into the feasibility of inband full-duplex self-backhauling. Overall, it can be concluded that a self-backhauling access node can greatly benefit from the inband full-duplex capability but the downlink and uplink traffic ratio should be quite symmetric.

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