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Motivation of this document
This document covers Part 4 of 4 of IDE4L Deliverable D6.1. It represents a study on the dynamic conditions in distribution systems. In particular the highlight has been the effect of PV systems on the grid dynamics. The main motivation of this work is given in IDE4L task 6.2. description of work, where the following assignment was included:

“[...] LV power grid modeling to analyze PV integration will be considered, investigating the effects of PV inverters in the grid, based on real profiles of generation, and analyze the impact on aspects such as unbalance on single-phase grids, minimization of DC current components, voltage fluctuations and harmonic distortion.”

The document concentrates in particular on the effect of PV in current distribution grids, particularly on the dynamics of voltage, like voltage fluctuations. The document is organized as follows. Sections 1 and 2 set up the scene by means of reviewing current challenges in distribution grids together with an in-depth literature review on dynamic conditions in distribution systems. A review on general power quality issue on distribution is presented, providing afterwards a more detailed analysis of the dynamic voltage behaviour. In section 3, grid benchmarks to be used in the simulations are described. In section 4, the small signal stability analysis is modelled in order to represent the dynamic behaviour of the system. Eventually in section 5 the stability analysis on A2A grid is described.
1. Challenges in current distribution grids

Recently, PhotoVoltaic (PV) Distributed Generation (DG) is spreading quickly within the electric distribution systems worldwide, and its diffusion is the fastest-growing among the renewable energy sources [1]. As this penetration increases, there are some issues that can appear in the electric grid [1–3] and, in particular, at lower levels of the distribution network, Medium Voltage (MV) and especially Low Voltage (LV) sides, because the largest part of Distributed Energy Resources (DERs) is connected here. PV-DG may be generically classified into three types according to [1]:

- utility-scale PV-DG, with Megawatt sizes;
- medium-scale PV-DG, with capacities from 10 kW to 1 MW that includes installations on small or large buildings;
- small-scale PV-DG, with capacities up to 10 kW;

and from [3], about 70% of the installed PV capacity is located at the LV level. So PV generation mainly consists of distributed small-scale rooftop units that are installed at customer side and connected to secondary lines, at 240 V for Europe connections [1]. This part of the grid was not originally designed to host generation units and this may lead to some problems. In Fig. 1.1, we can see the fast spreading of PV DERs within the power system and also at which level they are usually connected.

Among the issues that can be caused by a high DG penetration, there are the increasing of feeder voltages and the voltage flicker. The first one may be caused by a large DG penetration, especially when the power generation is high and the load demand is low. The second one is due to the intermittent nature of renewable energy sources. Considering PV units, their output powers strongly relies on the solar irradiance that can vary rapidly, for example when the clouds move fast. These two issues are particularly evident when large PV-DG plants are connected close to the end of long feeders. Furthermore, DG changes the voltage profiles making them not more monotonically decreasing along the feeder and it can increase the unbalances, since the greatest part of PV units is single-phase [1,2].

Voltage fluctuations, caused by the intermittence, impact on the voltage regulation devices, usually load tap chargers, which may experience a greater number of operations per year. The lifetime of these devices, and in general of all automatic line equipment, such as capacitors and voltage regulators, can be reduced by the increasing DG penetration, requiring more maintenance [1, 2]. Intermittence causes also frequent switching of voltage-controlled capacitor banks and frequent operation of voltage regulators, leading to reactive power flow fluctuations. If these variations are large enough, this may also affect subtransmission and transmission systems, with possible important economic impacts, because the transmission of reactive power is more expensive than its local supplying [1].

In high penetrated DG scenarios, another possible issue is the reverse power flow on the feeder, when the local DG exceeds the local load demand. This phenomenon can negatively affect protection systems, since they are usually designed for unidirectional power flows (e.g. over-current protection) [1,3]. Finally, also power losses of the grid are affected by DG: while low and moderate DG penetration levels can help reduce the losses, too high penetration levels could increase them [1].

PV generators are interfaced to the grid by power electronic converters, i.e. inverters, which can improve the operation of the grid thanks to their control capabilities. For example, inverters can support the voltage, regulate the Power Factor (PF), and balance the currents on each phase [2]. With other words, exploiting active and reactive power control capabilities of inverters, it is possible to decrease their penetration impact, alleviating the problems we described before [3].

Nowadays, the small-scale DERs are supposed to provide all the available active power at unitary PF: they operate as constant current generators that inject the current in phase with the voltage itself [4]. This is imposed by some standards, for instance [5], that do not allow the inverters to inject reactive power [2]. However, allowing the inverters to inject reactive power, and so to operate at non-unity PF, can help reduce voltage fluctuations caused by intermittence of renewables and to reduce the steady-state voltage rise [1-3]. Reducing the fluctuations can minimize the impact of PV on voltage regulators and switched capacitors extending
their life cycle, while steady-state voltage support can help accommodate more DERs in the grid [2].

DG can increase frequency variations of the electrical system, for example it can cause over-frequencies when there is a surplus of generation [3]. In such scenario, a sudden disconnection of a large share of the PV generation capacity can cause severe under-frequencies and even rolling blackouts. New interconnection requirements are trying to provide smoother responses to frequency variations of PV systems, when system over-frequencies appear [3,6,7]. Also power curtailment during an over-voltage event can help stabilize the system operation [2].

As the penetration of PV and in general of DERs increases, it becomes more and more important to study the system performances to understand if the grid is able to host additional DERs, keeping a proper voltage quality at all nodes. Indeed, the intermittence can negatively affect voltage control, power quality and the operation of the electrical system, as described so far. These effects may be caused by the potential interaction of all the devices of the grid, such as generators, loads, transformers, voltage regulators, etc.; and for this reason analysis and simulation of the system cannot be only static or quasi-static, but they have also to be dynamic [1,2].

Dynamic studies in this research area usually consider the effects of transient phenomena caused by the DERs, as the intermittence, or caused by special events, as faults and islanding conditions. The time frame for these analyses may vary from sub-second time steps to several hours: for instance PV inverters have dynamics of sub-second time step, automatic voltage control devices have dynamic of sub-minute or minute time steps, and loads can have longer dynamics [1].

2. Bibliography review

2.1 Introduction to the dynamics study

Different types of energy source and several kinds of load make the distribution grid scenario very complex: this aspect is also accentuated by the microgrid management algorithms, because all the devices in the grid can potentially act at the same time and with different time scale behaviors. There are low level controllers for the DG units, high level energy market mechanisms or algorithms that improve the quality of service at intermediate level and many others. This complexity and the time scale separation
suggest a layered architecture, as shown in Fig. 2.1 [8, 9]. Tertiary control usually dispatches active power generation on the basis of economic reasons and so it has the slowest dynamics. Secondary control algorithms take care of these power references while optimizing other aspects of the microgrid, for example improving the power quality, the stability, reducing the losses of the grid and so on. The lower level of control, that is the primary control, is usually of local type (at single inverter/device side) and requires very little or no communication between different units and it acts on very fast dynamics [8, 9].

Voltage stability and dynamic analysis for electric systems with a high DG penetration are usually evaluated and performed for transmission levels, but few works focus on small-area LV grids [10]. Instead, the impacts of DERs on the distribution systems are often studied only by using steady-state analyses, rather than using dynamic tools [11]. Static analyses can be used to evaluate the steady-state solutions of the system, understanding for example how the power losses of the grid change as the

![Figure 2.1: A possible layered architecture for the simultaneous execution of different algorithms in a smart microgrid [8]](image)

DG increases or how to control the DERs to reduce these losses. Other static studies can account the feeder capacity to host additional DERs or they can evaluate how to control the DERs to improve the steady-state voltage profile along the feeders. Moreover, a steady-state approach enables the study of protection systems and how they should be changed according to the DG penetration and also the study of unbalances [11, 12].

Since this complex scenario could be difficult to be addressed via analytic analyses in terms of dynamic, several works use simulation approaches. These types of study can provide insights on why it is important to consider the dynamic behavior of a distribution grid, rather than considering only its steady-state solutions. Dynamic analyses, both via simulation and analytic approach, can describe the interactions of the multitude of players that are involved in the electric system, with different degrees of approximation. This enables the understanding of potential dangerous instabilities and also the design of monitoring or control architectures for the future grids. In the next section, some papers that propose simulation studies are reviewed to show the important dynamic behaviors that cannot be neglected anymore in the distribution power system analysis.
2.2 Simulation approach

Nowadays, computation capabilities are very powerful and so they allow simulating very complex and extended scenarios. These capabilities have pushed different works to propose several types of simulation scenario for distribution grids. A great part of these proposals simulation analyses for islanded (or autonomous) operation of microgrids, because here there are great dynamic concerns about the grid operation. Among these, a large number is about the P − f and Q − V droop control [13, 14]. The simulations of this set of papers can be very complex and extended, because they usually adopt very detailed models, for example including primary and secondary level controllers of inverters also for very large grids: for instance [15] simulates an autonomous distribution network of 200 consumers.

The goals of this report are only about the grid-connected operation and so the models and results that we will propose in the following chapters regard only this mode of operation. Also this operation mode has been considered so far: for example [16] studies the impacts of DERs that are interfaced to the distribution grid via induction generators, synchronous generators and inverters. This work shows the appearing of oscillations on the voltages of the grid during some transients, depending on the interface device and the DG penetration level. Also the effects of some control techniques to improve the power quality are investigated. In this work, all the loads are modeled as static and so their dynamic effects are not considered. A similar approach is described by [17, 18], which consider synchronous generators and inverter interfaced DERs and compare them in terms of transient stability. The simulations of [17] analyze the behavior of the system applying a three-phase fault on the transmission side of the benchmark. Among the results, this paper shows that converter-connected DG increases the maximum rotor speed deviations of the synchronous generator and this is a signal that the system is becoming more unstable. This instability is due to the reduction of the total inertia, and so of the stored kinetic energy, that is caused by the substitution of rotating generators with inverter interfaced generators [17]. Some simulations of a transmission grid are performed in [18] and PV DERs interfaced by inverters are considered and modeled in detail with their Maximum Power Point Tracking (MPPT) algorithm. Via simulation, this work describes how the stability changes according to the DG penetration level. Some example results of this analysis can be found in Fig. 2.2, where we can see that the frequency and the voltage deviations at one node of the grid increase as the DG penetration increases (after a transient of PV plant disconnection). It is important to highlight that [18] simulates a transmission grid, showing the interactions among traditional synchronous generators and inverter interfaced PV generators.

Several studies on voltage stability of electric power systems with high PV penetration are done for transmission grids and very few consider MV or LV grids [10]. Distributions systems have quite important differences compared to higher voltage levels, that are low X/R ratio cables, long tap switching delays, small PV units, etc., and so the study of the PV integration impacts has to be focused on the particular level of the power system. The IEEE 13 bus system has been modified in [10] to consider a MV scenario and it has been extended to analyze the impacts of the PV generation on the network stability. The simulation model of the grid in [10] is very rich and detailed and it considers static and dynamic (induction motor) loads, transformers, circuit breakers, voltage regulators (e.g. tap chargers), unbalances, thermostatic loads, etc. The inverter model for the PV DERs includes the MPPT algorithm and the current regulation loop, while the paper addresses the cloud effects on the voltage stability and intermittency. The paper shows that the grid can become unstable when it is heavily loaded and the PV DERs inject active power at unitary PF: this instability can be seen with 40% of DG penetration level and it is caused by the voltage fluctuations induced by cloud movements. The importance of dynamic modeling for the loads is also highlighted by [10], since some instability problems cannot be seen only with static load models. Some approaches to mitigate the voltage fluctuations, as involving electric storages and injecting reactive power, are also studied and analyzed. More details on these control topics will be shown in Sec. 2.6. Some results of this paper are summarized in Tab. 2.1. Simulations of a large power system from the transmission level down to the distribution level (HV, MV and LV levels), where the DERs are connected at the LV side, are performed in [19], while a similar scenario is considered in [20], where a Real-Time Digital Simulator (RTDS) is used.
Table 2.1: Voltage status at one bus with different PV penetration levels, load models and voltage support strategies [10]

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<th>Load</th>
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<tr>
<td>Dynamic</td>
<td>0%</td>
<td>Stable</td>
</tr>
<tr>
<td>Dynamic</td>
<td>10%</td>
<td>Stable</td>
</tr>
<tr>
<td>Dynamic</td>
<td>20%</td>
<td>Stable</td>
</tr>
<tr>
<td>Dynamic</td>
<td>30%</td>
<td>Stable</td>
</tr>
<tr>
<td>Dynamic</td>
<td>40%</td>
<td>Stable</td>
</tr>
<tr>
<td>Static</td>
<td>40%</td>
<td>Stable</td>
</tr>
<tr>
<td>Dynamic</td>
<td>40% PV + 20% Storage (Bus 650)</td>
<td>Unstable</td>
</tr>
<tr>
<td>Dynamic</td>
<td>40% PV + 20% Storage (Bus 632)</td>
<td>Stable</td>
</tr>
<tr>
<td>Dynamic</td>
<td>40% PV + PV reactive support</td>
<td>Stable</td>
</tr>
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This starting works have been reported to show the importance of the dynamic analysis in distribution grids that are populated by a great number of DG units. In particular, some concerns and issues that can be problematic and dangerous, as the DG increases, can be highlighted only with dynamic simulations or analyses.

Figure 2.2: Simulation results from [18]: on the left the frequency and on the right the voltage transient when a PV plant is disconnected; different results for different DG penetration levels

2.3 Q-V modal analysis

Classical power system analysis includes the identification of the operating points of the system where significant changes occur in the voltage stability [21]. In this section, a classical approach used in power system theory is described to assess the stability of voltage. The approach that is described in [22] will be used.

The relationships between network node voltages and currents of a grid can be expressed by node equations and all these relations can be written with the node admittance matrix as follows [22]:
n is the number of the nodes of the grid

$V_i$ is the complex phasor of the voltage (to ground) at node $i$

$I_i$ is the complex phasor of the current injected to the network at node $i$

$Y_{ii}$ is the sum of all the complex admittances connected at node $i$

$Y_{ij}$ is the opposite of the sum of all complex admittances between nodes $i$ and $j$

and the bar symbol $\overline{\cdot}$ refers in general to an entity that is represented by its complex phasor, while the dot symbol $\cdot$ refers to a simple complex entity (that is not a phasor).

Collecting in the vector $\bar{V}$ all the node voltages $\bar{V}_i$ for $i = 1, \ldots, n$ and in the vector $\bar{I}$ all the node currents $\bar{I}_i$ for $i = 1, \ldots, n$, in general an electric power system can be dynamically modeled with a differential algebraic equation system [22]:

\[
\begin{align*}
\frac{dx}{dt} &= f(x, V) \\
I &= g(x, V)
\end{align*}
\] (2.2)

Where $x$ is a vector (may be complex) that collects the state variables of all the individual devices connected to the nodes of the grid and $f()$ and $g()$ are two generic non-linear functions. However, with (1) we saw that the second equation of (2) can be written as $\bar{I} = \bar{Y}\bar{V}$, where $\bar{Y}$ is node admittance matrix. The power flow equations can be written as:

\[
\dot{S}_k = P_k + jQ_k = V_k I_k^* \quad \forall \quad k = 1, \ldots, n
\] (2.3)

Where $(\cdot)^*$ operation refers to the conjugation of complex numbers and $\dot{S}_k$ is the complex power injected at the node $k$. From Eq. (1):

\[
\bar{I}_k = \sum_{m=1}^{n} \dot{Y}_{km} \bar{V}_m
\] (2.4)

And substituting this equation in Eq. (2.3), we obtain:

\[
P_k + jQ_k = \bar{V}_k \sum_{m=1}^{n} \dot{Y}_{km}^* \bar{V}_m
\] (2.5)
Writing the voltages with their polar representation \( V_k = V_k e^{j\theta_k} \) and the admittances with their real and imaginary parts \( Y = G_{km} + jB_{km} \), Eq. (2.5) becomes:

\[
\begin{align*}
P_k &= V_k \sum_{m=1}^{n} \left( G_{km} V_m \cos(\theta_k - \theta_m) + B_{km} V_m \sin(\theta_k - \theta_m) \right) \\
Q_k &= V_k \sum_{m=1}^{n} \left( G_{km} V_m \sin(\theta_k - \theta_m) - B_{km} V_m \cos(\theta_k - \theta_m) \right)
\end{align*}
\]  

(2.6)

Thus, \( P_k \) and \( Q_k \) at each bus \( k \) are functions of all the voltage magnitudes \( V_i \) (for \( i = 1,..., n \)) and angles \( \theta_i \) (for \( i = 1,..., n \)) of all buses.

Eq. (2.6) is the starting point of lots of load flow problem formulations and they are also the starting point for the Q-V modal analysis [22]. In particular, we have to obtain their linearized version, that is:

\[
\begin{bmatrix}
\dot{P} \\
\dot{Q}
\end{bmatrix} =
\begin{bmatrix}
J_P & J_P V \\
J_Q & J_Q V
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\dot{V}
\end{bmatrix}
\]  

(2.7)

Where

- \( \dot{P} \) is the vector of incremental changes in the bus active powers
- \( \dot{Q} \) is the vector of incremental changes in the bus reactive powers
- \( \dot{\theta} \) is the vector of incremental changes in the bus voltage phases
- \( \dot{V} \) is the vector of incremental changes in the bus voltage amplitudes
- \( J \) is the Jacobian matrix

The matrix \( J \) is very important in power flow problems and it is the same that is used for the Newton-Raphson method [22].

System voltage stability is affected both by \( P \) and \( Q \), however at each operating point we can consider a constant \( P \) value and evaluate the voltage stability from the reduced relationship between \( Q \) and \( V \) [22]. So, setting \( \dot{P} = 0 \) in (2.7), it follows that:

\[
\dot{Q} = J_R \dot{V}
\]

where

\[
J_R = J_{QV} - J_{Q\theta} (J_{P\theta})^{-1} J_{PV}
\]  

(2.8)

Where \( J_R \) is the reduced Jacobian matrix of the system. To relate the voltage variation as a function of the reactive power variations, we may write:

\[
\dot{V} = J_R^{-1} \dot{Q}
\]  

(2.9)

The \( J_R \) is the reduced V-Q Jacobian and its \( i \)-th diagonal element is the V-Q sensitivity at bus \( i \). The V - Q sensitivity at a bus represents the slope of the Q - V curve (V as function of Q) at the given operating point. A positive V - Q sensitivity is indicative of a stable operation; the smaller the sensitivity, the more stable system. On the other hand, a negative V - Q sensitivity indicates an unstable operation [22]. Voltage stability characteristics can be identified by computing the eigenvalues of \( J_R \) [23, 24], for this reason let us write:
\[ J_R = \xi \Lambda \mu \]  

(2.10)

where
- $\xi$ is the right eigenvector matrix of $JR$
- $\mu$ is the left eigenvector matrix of $JR$
- $\Lambda$ is the diagonal eigenvalue matrix of $JR$

From Eq. (2.10), $J_R^{-1} = \xi \Lambda^{-1}$ and substituting it into Eq. (2.9):

\[ V = \xi A^{-1} \mu \hat{Q} \quad \text{or} \quad V = \sum \frac{\xi \mu_i}{\lambda_i} \] 

(2.11)

Where $\xi_i$ is the i-th column of $\xi$, $\mu_i$ is the i-th row of $\mu$ and $\lambda_i$ is the i-th eigenvalue, that is the i-th element of the matrix $\Lambda$. If $\lambda_i$ eigenvalues are positive, the system is voltage stable and, in this situation, the magnitude of $\lambda_i$ determines the degree of stability of the i-th modal voltage: the smaller the magnitude of a positive $\lambda_i$, the closer the i-th modal voltage is to being unstable. On the other hand, if $\lambda_i$ eigenvalues are negative the system becomes unstable [22]. These results come from the loadability limit and bifurcation analysis: usually when the Jacobian $JR$ becomes singular a saddle-node bifurcation on the dynamic model of the power system, for example Eq. (2.2), appears. In this point, the conditions of the stability of the system change. More details on these mathematical aspects can be found in [21]. This analysis can be performed easily and without any knowledge about what devices are connected at each node, because it considers only the electric network constrains, i.e. Eq. (2.1), to study the existence of the power flow solution and its stability. However, this method does not consider the dynamic behaviors of the different players that can be found in a distribution grid, that are generation units, loads and control devices.

### 2.4 Components’ dynamic modeling

At the beginning of this chapter, we anticipated that the dynamic study of distribution grids can be very complex and varied due to the large amount of different devices here connected. In this section, we introduce some papers that provide a classification of these devices together with their mathematical models, which can be used for both simulation studies and analytic analyses.

#### 2.4.1 DER modeling

In the LV or MV distribution grids, different DER types can be connected, as for example wind energy resources interfaced by induction generators, PV sources interfaced by inverters, batteries and energy storages, gas microturbines, diesel generators, fuels cells, etc. Even if the generation devices are several and different, well-established models have already been proposed for all of them [25]. Several works present surveys on DER modeling, also with different levels of approximation. For instance, [25] is a very complete survey of DG devices and it provides several references that describe the mathematical models for these players. Models of gas microturbines, fuel cells, PV systems and wind turbines are described in [26] and some Simulink models are provided too. Moreover, simplified models of the power electronic interfaces for the DERs are described. Dynamic simulation models for converter-connected DC sources (e.g. fuel cell, PV also with MPPT algorithm), doubly fed induction generators for wind turbines, converter-connected synchronous generators, etc. are provided by [27,28]. Finally, [29] focuses on wind energy providing dynamic models for wind power generation, including the wind turbine model, the doubly-fed induction generator model and also the
models of their controllers.
So for each DG unit, a well-known model exists and also approximated models have been proposed by several works to reduce the complexity of large grid analyses. Very detailed model are often used to analyze primary level controllers, while approximated models can be used to study higher control levels, for example the secondary level [8].

2.4.2 Load modeling

An electric power system is composed by power generators, electric lines and loads, together with other control devices. Among these, the load representation has remained the least accurate of these components, even if the load models can play a significant role on the power system dynamic behavior [30, 31]. Therefore, accurate load models are needed to evaluate the dynamic characteristic of the grid and to understand its instability issues. A load bus usually consists of a large number of different load types (motor, lighting and electrical appliances) and modeling every load component may not be practical and interesting. Instead, appropriate aggregated load models can capture the significant part of the overall load behaviors and they can be practically feasible [31]. However, while modeling a single device is easy, the task of finding an appropriate aggregated model for lots of loads is not easy and without approximations.

To develop a load model we have to determine a suitable model structure, first, and then evaluate the model parameters. For example [31] proposes the following dynamic load model:

\[
\begin{align*}
T_p \frac{dx_p(t)}{dt} & = -x_p(t) + P_0 \left[ \frac{V(t)}{V_0} \right]^{N_{pr}} - \left[ \frac{V(t)}{V_0} \right]^{N_{pt}} \\
\frac{P_d(t)}{x_p} & = x_p(t) + P_0 \left[ \frac{V(t)}{V_0} \right]^{N_{pt}} \\
T_q \frac{dx_q(t)}{dt} & = -x_q(t) + Q_0 \left[ \frac{V(t)}{V_0} \right]^{N_{qr}} - Q_0 \left[ \frac{V(t)}{V_0} \right]^{N_{qt}} \\
\frac{Q_d(t)}{x_q} & = x_q(t) + Q_0 \left[ \frac{V(t)}{V_0} \right]^{N_{qt}} 
\end{align*}
\]

(2.12)

Where \( V(t) \) is the instantaneous load voltage amplitude, \( x_p \) and \( x_q \) are load states for active and reactive power respectively, \( T_p \) and \( T_q \) are time constants, \( P_d \) and \( Q_d \) are the active and reactive load power demands, and \( P_0 \), \( Q_0 \), and \( V_0 \) denote nominal active, reactive power and voltage respectively, and are assumed to be known. The exponents \( N_{pr} \), \( N_{pq} \), \( N_{pt} \) and \( N_{qt} \) stand for steady-state and transient load-voltage dependencies and they have to be estimated to match the actual load characteristics.

Once the model of the load is chosen, indeed, its parameters have to be estimated and this is usually done following two approaches: the component-based approach and the measurement-based approach [32]. A component-based approach consists of deriving the model parameters by aggregating models of the individual components, while the measurement-based approach is used to estimate the parameters of the model starting from real measures of the bus. The last approach can capture the actual load behavior during system disturbances so that accurate parameter values can be obtained [31]. Several works have studied and proposed methods to estimate these parameters [33].
The greatest part of works, that propose a dynamic aggregated model, considers the parallel connection of a static and a dynamic load. The static part is usually composed by a constant impedance $Z$, constant current $I$ and constant power $P$ part, ZIP load model, whereas the dynamic part is usually described with an induction motor load (Fig. 2.3). This composite load model is the most attractive one in power system modeling area because of its clear physical meaning and high proportions of induction motors in the grid [33]. Following [34], the static part is:

$$\begin{align*}
P &= P_0 \left[ P_Z \left( \frac{V}{V_0} \right)^2 + P_I \left( \frac{V}{V_0} \right) + P_P \right] \\
Q &= Q_0 \left[ Q_Z \left( \frac{V}{V_0} \right)^2 + Q_I \left( \frac{V}{V_0} \right) + Q_P \right]
\end{align*}$$

Where $P_0 + P_I + P_P = 1$ and $Q_0 + Q_I + Q_P = 1$. In these equations, $P_Z$ and $Q_Z$ are the active and reactive powers of the constant impedance load, $P_I$ and $Q_I$ are the active and reactive powers of the constant current load, $P_P$ and $Q_P$ are the active and reactive powers of the constant power load, $P_0$ and $Q_0$ are the active and reactive powers absorbed by the ZIP load at the nominal voltage $V_0$. The dynamic model of the induction motor is described as:

$$\begin{align*}
\frac{dE}{dt} &= -jsT_s \hat{E} - \hat{E} + j(X - X') \hat{I} \\
\frac{ds}{dt} &= T_m - T_c \\
\hat{V} &= \hat{E} + (r_s + jX') \hat{I}
\end{align*}$$

Where
\[ X = X_s + X_m \]
\[ X' = x_g + \left( \frac{1}{X_m} + \frac{1}{X_r} \right)^{-1} \]
\[ T' = \frac{T_r + X_m}{R_r} \]
\[ T_e = \text{Re} \left( E I^* \right) \]
\[ T_m = T_0 \left( A \omega^2 + B \omega + C \right) \]
\[ 1 = A \omega_0^2 + b \omega_0 + C \]

And where:

- \( R_s \) is the stator winding resistance (in p.u.)
- \( X_s \) is the stator leakage reactance (in p.u.)
- \( X_m \) is the magnetizing reactance (in p.u.)
- \( R_r \) is the rotor resistance (in p.u.)
- \( X_r \) is the rotor leakage reactance (in p.u.)
- \( T_j \) is the rotor inertia constant
- \( A; B \) are the per unit components of motor torque

but also other works use a similar approach \([32, 33, 35, 36]\). The work \([37]\) provides a further approximation by an order reduction for the induction motor load in Fig. 2.3 for dynamic analysis. Several variants exist for the static part of the load model in Fig. 2.3. For example, \([32]\) considers also an exponential load instead of the ZIP load and so the following model replaces the one in Eq. \((2.16)\)

\[
\begin{align*}
P &= P_0 \left( \frac{U}{U_0} \right)^{k_{pu}} \\
Q &= Q_0 \left( \frac{U}{U_0} \right)^{k_{qv}}
\end{align*}
\]

where \( k_{pu} \) and \( k_{qv} \) are real constants. Further extensions of the static part are presented in \([38]\), considering also the frequency dependencies of the load. However, in a small LV/MV grid-connected network these variations are usually very small and negligible: frequency variations are more important for example in the autonomous operation of the grid or in large power systems (see the next section).

This section has presented some dynamic load modeling approaches, however in the dynamic studies of large distribution systems as a whole, usually the load dynamics are not considered. This will be seen in the next section, where some analytic approaches are presented.

### 2.5 Dynamic analysis

Among the analytic studies of the grid stability, we review two different approaches, one consists on a very detailed method without or with small approximations and another approximated approach. We are going to pay attention on the load modeling choices of these works, completing what we said in the previous section.
2.5.1 Modeling of conventional generator-based grids

Stability analysis for power system usually focuses on the interactions of conventional generators, i.e. rotating machines, often for the transmission levels of the grid. This has pushed some works to extend these results also to the distribution grid, as we will see in this section. However they usually do not include inverter-interfaced DERs.

Among these works, the paper [39] considers rotating machines as steam-turbine-generator, hydro-turbine-generator, combustion-turbine-generator and wind turbine with induction generator, but no power electronic interfaced device is included. This work describes a set of dynamic models for the generators and also some approximations of these models via order reduction and then linearization. A framework to link these models is then proposed in [39], enabling the stability analysis of the grid through a small-signal approach and so an eigenvalue analysis.

Generally in distribution systems, we cannot assume that the reactive power flow depends mostly on the voltage amplitude and the active power flow depends mostly on the phase of the voltage, because the electric lines have strong resistive components. So if the primary control of the DG rotating machines is designed with decoupled power dependencies, frequency and voltage instabilities may occur, as pointed out by [40, 41]. Furthermore, [42] shows through a dynamic analysis based on wind energy distributed generators, that instabilities could arise in the interconnected grid if each DER is only designed as a stable standalone device.

Figure 2.4: Schematic of the IEEE 23 kV 30 node distribution system with new DGs [41]
With other words, component-wise stability does not ensure stability of the system as a whole, and this confirms the importance of studying the dynamic characteristics of a grid instead of focusing on a single device. In these works, coupled dependencies for the power flows are considered to assess the voltage and frequency stability and the used models also include the dynamic description of frequency variations, since rotating machines are considered as interfaces for DG units.

The stability analysis is addressed in these papers with a small-signal model that has been obtained through linearization and thus it allows and eigenvalue analysis \([40–42]\). The network model that links all the DERs is expressed as static power constraints and so it is linearized, as in Eq. (2.6) and then Eq. (2.7). However, dynamic loads and PV generation interfaced by inverter are not included in these models. Concerning the test-cases for the model validation, \([40, 41]\) use the IEEE 30-node distribution system, with a couple of DG electric machines, as Fig. 2.4 shows, and the power system of Flores Island (Azores Archipelago, Portugal), as Fig. 2.5 shows.

An object-oriented power system simulation environment in Matlab/Simulink is developed and presented by \([43]\). Therein, the loads are modeled as static constant impedance and constant power loads, but also a dynamic load model is provided. In this framework, this is the first work that considers the dynamic behaviors of the loads. They are described as admittances \(G + jB\), with the conductance \(G\) and susceptance \(B\) that varies according to this differential system:

\[
\begin{align*}
\frac{dG}{dt} &= \frac{1}{\tau} (P_{ref} - P) \\
\frac{dB}{dt} &= \frac{1}{\tau} (Q_{ref} - Q)
\end{align*}
\]

(2.17)

here \(\tau\) is a time constant, \(P_{ref}\) and \(Q_{ref}\) are the desired constant active and reactive power respectively for the load, \(P\) and \(Q\) are the
instantaneous active and reactive powers of the load. Anyway, no physical meanings are provided about this modeling choice. This dynamic load model extends that in [44], where a unity PF load is considered for simplicity. According to [44], a constant power load cannot change instantaneously from one demand level to another as the demand changes. So after a demand change, the load will first change according to its instantaneous characteristic that could be constant impedance or constant current load, and then it will adjust the current drawn from the system to match its power reference.

The transmission line dynamics are normally neglected in power system analysis and so the greatest part of the works that address dynamic and stability issues considers static representation of the grid [45]. This means that the currents and the voltages of the grid are modeled with the classic phasors and the grid is described with the static impedances. On the other hand, other approaches exist: for example, the paper [45] uses the dynamic phasors to model the electric lines, thus enabling a precise description of current and voltage dynamics. This work proposes some comparisons of load modeling in terms of stability region: considering an ideal voltage source, an electric line and some different types of load, such as constant impedance load, constant current load and dynamic load as in [44] and then as Eq. (2.17). This simple test-case is shown in Fig. 2.6. For these test-cases, the stability region is analyzed and compared using dynamic phasor representation [45]. Similar tests are then performed on larger systems that consist of an ideal voltage source (slack bus), a sixth-order two-axis model of a rotating generator and a load (see Fig. 2.7). Based on these analyses, the paper founds that a constant impedance load model does not cause any instabilities within the grid, but the use of a constant current model reduces the stability. Dynamic load models, according to their time constants, may exhibit similar behaviors and so they have to be accounted in a dynamic analysis of the power system.

![Diagram](image)

**Figure 2.7: Final test-case for the dynamic considerations used in [45]**

### 2.5.2 Detailed approach

The works presented in the previous sections do not consider DERs that are interfaced by power electronic converters, for example PV units. This lack is overcome by the papers that are presented in this section. However, lots of these works focus on the autonomous operation and in particular on droop control techniques, like in Sec. 2.2, and they often study the stability of the grid via linearized state-space models. The very detailed analysis of this set of works usually makes its generalization and its extension to larger grids difficult: the analysis is usually highly linked to the particular test-bed that has been considered, which is usually limited to few DERs.

The interactions among inverter interfaced DERs and conventional rotating generators are investigated by [46, 47], which propose small-signal models. The connection to the higher level of the grid is often modeled as an ideal voltage source with constant frequency (infinite bus), because a small part of distribution grid has small power generation and absorption capabilities and so it cannot influence the frequency of a large system. However in autonomous mode, the connection to a large system lacks and the devices of such grids have comparable power ratings and so they can originate frequency variations. For these reasons, in islanded grids the frequency variations have to be considered within the mathematical model, and this is usually done by papers that deal
with such problems [46, 47]. The approach of these works is very detailed and it allows designing the controllers of the microgrid down to the primary level. One drawback of this modeling detail is that the considered test-cases are usually quite small: for example, the grid of [47] in Fig. 2.8. These papers describe all the currents and the voltages of the grid with the dq0 or Park transformation [22], considering only linear loads (impedances), and then the model is linearized.

Similar approaches are adopted by [48,49], which develop small-signal models of microgrids containing the most-utilized types of power interface devices for DERs: synchronous generators, asynchronous generators and inverters are modeled in a very precise way considering also all the dynamics of their controllers. The small-signal model of each device is established separately and then the global model is set up in a global reference axil frame, allowing an eigenvalue analysis of the system. In [48], all the loads are modeled as a resistance-inductance series and the grid benchmark is still very small. On the other hand, the models of the electric network are without approximations in [47, 48], this means

![Figure 2.8: Test-bed for the small-signal analysis in [47]](image)

![Figure 2.9: Inverter modeling example from [49]](image)

that both these works describe the RL cables with the dynamics of inductor currents, rather than with static complex impedances. Another paper, which provides a very detailed modeling, considering droop control for islanded operation is [50].

In Fig. 2.9 there is a standard modeling example for the inverters of this set of works to show the usual level of detail that is assumed. Differently from the previous works, this paper recognizes the importance of considering also the dynamic behavior of the loads, in particular for the MV distribution grid. For this reason, in its test-case an induction motor load is included and modeled. Similarly, the paper [51] introduces the analysis of a dynamic constant power load that is interfaced to the LV grid via an active rectifier. This load belongs to the category of active loads, which are devices such as machine drives, back-to-back converter configurations, and consumer electronics with unity PF correction.
All these works build a state-space model to study the dynamics of the variations of the voltages, currents and powers close to a particular operating point (small-signal analysis). These models can be used to design the controllers, for instance primary or secondary ones, or to obtain the eigenvalues of the system. This eigenvalue analysis describes such variations and an example result is shown in Fig. 2.10, where the dynamics of different devices are identified. Small-signal models allow also performing some sensitivity analyses: the eigenvalue changes, and so the dynamic behavior of the system, can be related and analyzed according to some particular variations of some parameters, e.g. the DG penetration level. An example of this study is reported in Fig. 2.11.

![Figure 2.10: Results of the small-signal analysis in terms of eigenvalues [51]](image1)

![Figure 2.11: Results of the small-signal analysis in term of sensitivity of eigenvalues [51]](image2)

The set of papers described in this section has some common characteristics. One is that they usually describe all the dynamics down to the primary level control for the DERs resulting in a very complicated analysis and they usually are proposed for droop control of islanded grids. This makes difficult the generalization of these approaches to larger grids, also because the dimensions of the small-signal models become very large. Furthermore, no particular details are usually given to the load modeling, as anticipated also in the previous sections. Anyway, we saw that the most used approach for the stability analysis of distribution grids is the small-signal state-space model that is obtained through a linearization process. The paper [52] is slightly different, because it considers...
grid-connected operation of inverters, which are described with active and reactive power controllers. This approach is based on graph theory to build the matrices of the state-space model and it is modular and can easily be built for generic grids. Another general approach is proposed by [41].

Figure 2.12: Inverter controller in the dynamic analysis of [53]

Figure 2.13: Branch model in the dynamic analysis of [53]

### 2.5.3 Approximated approach

Referring to the control architecture of Fig. 2.1, the goals of this research activity are related to the secondary level control and its dynamics. For this reason, it is not necessary to use very detailed approaches, as those of the previous section, that describe also the primary controllers. Furthermore, including these aspects in an analysis of a large distribution grid can uselessly increase the hurdle of the mathematical model, resulting in a less handleable and meaningful tool. Simplicity is also required to make the method scalable and adaptable for grids with different sizes (also very large ones). Our attention has to focus on the dynamic behavior of a distribution grid that is an aggregation of DERs, but also loads (as we pointed out in Sec. 2.4.2) and where the interaction of a large amount of these units plays a significant role, rather than focusing on a single device. These aspects suggest us to use an approximated model for each device of the grid, for example exploiting the approaches of this section.

A scalable computational approach to microgrid modeling is given in [53]. This approach uses the automated state model generation algorithm to develop the microgrid model systematically and in an automatic way: this aspect makes it interesting. This model can be used to simulate the transients of the grid or to study small-signal stability, providing insights on the relationship between inverter controller parameters and system stability. However this work focuses on a particular local controller for DERs (shown in Fig. 2.12) for the autonomous operation of microgrids. This paper uses the line model of Fig. 2.13, while the benchmark microgrid is in Fig. 2.14: we can note the presence of transformers and linear loads (impedances). More interesting for the purposes of this report is the work [8] that shows a linear model that can help the design of a secondary level controller, proposing a model of the grid that relates the complex power references of the DERs to the voltage phasors.
In this work, all the nodes of the grid can contain either an inverter or a load and both are modeled with a first-order approximation:

\[
\frac{d\bar{I}}{dt} = -\bar{I} + \left(\frac{\dot{S}_{\text{ref}}}{V}\right)^* 
\]  

(2.18)

Where \(\bar{I}\) is the complex phasor of the node current, \(\bar{V}\) is the complex phasor of the node voltage, \(\bar{\tau}\) is the characteristic time constant and \(\dot{S}_{\text{ref}}\) is a complex power reference. In other words, the paper [8] approximates loads and inverters as constant power loads with a first-order dynamic of the current. Its mathematical tool keeps a simple description for each node of the grid, while it can study the interactions of a large number of these nodes: it is scalable. This paper obtains a linear dynamic model for the grid, enabling the eigenvalue analysis for dynamic studies or to design a secondary level control for DERs.

We are interested in developing a small-signal model that is scalable and with simplified descriptions for the devices of the grid, for example as in [8, 53]. Another important point to be considered is the secondary level algorithm of the grid that can have impacts on the voltage stability. Thus, on the next section some secondary controllers, which improve the grid operation, will be introduced.

## 2.6 PV-DG grid supporting

Inverters for DG systems may produce active and reactive power at any level, of course according to the active power supplied by the primary energy source and according to the rated power of the electronic device: this allows the DERs to produce any power at any PF [4, 54]. However, some standards, like [5], impose the injection of all the available active power at unity PF and this can be a concern. In such situation, the PF at the Point of Common Coupling (PCC) can go down on a lagging power system and it would be better if the inverters supply the reactive power locally, instead requiring it from the PCC [54]. As anticipated in Sec. 1.1, there are other benefits from allowing the inverters to regulate their active and reactive powers. For instance, they can support the voltage and frequency, improve the power quality and increase the hosting capacity of the grid [1–3, 54]. In this section, some voltage and frequency support techniques (in particular for PV systems) are shown to understand their potentiality and possible dynamic issues.

### 2.6.1 Voltage support

PV generators do not have any rotating parts, unlike conventional generators, and thus they do not have inertia. This can cause some problems when the solar irradiance at the PV changes rapidly, for instance due to cloud movements, because also the output...
power of the inverter changes rapidly. Fast power changes can cause voltage sags or dips and, in these situations, injecting reactive power can help mitigate the voltage variations [55, 56]. Inverters can regulate the voltage together with the conventional devices of a radial distribution system that are load-tap-changing transformers at substations, line-voltage regulators or switched capacitors on feeders [54].

Generally speaking, the local reactive power injection can pursue two different goals: the reduction of the voltage drop along the electric line and the minimization of the distribution losses. However, these two objectives are in competition and a trade-off has to be done [56]. With other words, the minimum for the power losses and the minimum for the voltage amplitude drop are achieved for different values of reactive power injection by inverters. Local control techniques that measure and act only on local quantities do not require additional communication or coordination infrastructures. This usually makes them more robust, but, on the other hand, local schemes act based on little information and so optimal operation in general could not be achieved. Centralized controllers exploit communication infrastructures and more information about the grid operation to perform better optimization. Some comparisons between these two approaches are described in [56].

One of the simplest voltage support techniques consists of injecting negative reactive power as the local voltage amplitude increases and generating a positive reactive power when it decreases [54, 56]. This technique has already been regulated and imposed by some country level standards, as for instance the Italian [6] and the German (see reference in [57]) ones. Referring to the Italian standard [6], each PV inverter with rated power greater than 6 kW has to generate reactive power according with the \( Q - V \) droop characteristic that is shown in Fig. 2.15. Many existing standards require this type of regulation, but many connection rules still do not specify any particular requirements on their dynamic response [57]. For example, the standard [6] only provides an upper bound on the transient time for this regulation: it states that the reactive power has to reach the steady-state value within 10 s. A similar curve is used in [58], which specifies that the target of this regulation is to limit the maximum voltage and not to keep the voltage to a specified level, as it happens for the reactive power control in HV systems. The work in [59] shows a real implementation of this controller together with a detailed design of the inner inverter control loops. \( Q - V \) droop control for inverter voltage support has been studied also by other works, as [57, 60–62], and among these [55] shows that this controller can reduce the voltage fluctuations due to cloud movements.

![Figure 2.15: Q–V droop characteristic for inverters with rated power greater than 6 kW according to Italian standard [6]](image)

LV grids usually have cables with X/R ratio less than one and so the voltage amplitude depends more on the active power flow than on the reactive power flow. This consideration has driven the work [63] to adopt a \( P - V \) droop characteristic to prevent over-
voltages caused by DG. So if the output voltage of the inverter increases over a certain threshold, the active power generation will be limited by a local controller.

More sophisticated solutions are based on centralized or distributed controllers: for instance, the paper [64] proposes a centralized approach to reduce the distribution losses that relies on communication infrastructure and centralized computation capabilities. Its control architecture can be seen in Fig. 2.16. A similar approach is proposed by [65] which presents a central controller that communicates with the DERs tanks to a narrow-band communication infrastructure. Optimal reactive power compensation is also the goal of [66], which describes a distributed control that minimizes the network losses while keeping the voltage magnitudes of all the nodes in a precise range, i.e. the one imposed by a particular standard. This method requires local communication and local knowledge of the network topology and state, while the optimization of the network losses and voltage profile are achieved regulating both active and reactive inverter powers in [67].

The greatest part of these works proposes algorithms to improve the steady-state operation of the grid, i.e. reducing power losses or voltage variations. For this reason, the validations of these methods are often only static: some simulations only show how the steady-state operation is improved. An example of these results and it can be seen in Fig. 2.17, also the paper [63] uses only static considerations to prove the operation of its controller. Only few papers consider the dynamic characteristics and the stability of the grid of the proposed algorithm [57]. Even if the aim of the voltage control is not to react on the fast voltage variations, but to compensate part of the voltage rise caused by the PV infeed, the stability of a large system with lots of these generators controlling the voltage must be guaranteed [57]. Thus, this topic is very important and it has to be investigated.

Figure 2.16: Decentralized controller architecture in [64]
IDE4L is a project co-funded by the European Commission

Figure 2.17: Example of static results to prove the performances of active/reactive power control to improve the voltage profile or to reduce the power losses: in this figure, the voltage profile along the feeder to compare different inverter controllers [67]

Regarding the papers that focus on Q − V droop control, the results usually aim to show that the voltage profile is more flat with this regulation, rather than evaluating the stability of the system with a lots of PV inverters that try to control the voltages [61]. Only few of them consider the dynamic stability of the proposed voltage support method, for instance, in [61] the stability is studied in terms of discrete system for the Q − V controller. However, this paper considers variable thresholds $Q_{\min}$ and $Q_{\max}$ of Fig. 2.15, differently from the standard [6], where they are constant. These limits are symmetric and recalculated at each time step.

Figure 2.18: Two-bus system: voltage magnitudes and reactive power injections from PV inverter for two types (A and B) of dynamic implementation of Q − V droop control from [61]

with a discrete-time transfer function: the paper compares the dynamic characteristics of two different types of these filters. Analyses and simulations of a 2 bus system and a 3 bus system are presented to understand how the stability is affected by the particular filtering strategy and by the time constants of such filters. For example, Fig. 2.18 shows the instabilities that can arise for the 2 bus system. These results are then extended for a grid with a generic number of inverters, however the dynamics of the inverters and of the loads are not considered therein. Both steady-state and dynamic considerations on the Q − V droop control for voltage support are provided with a more rigorous approach in [62], however also here the analysis does not consider the dynamic behavior of the inverters and of the loads.

The reference [57] proposes a simplified representation of the system to study the stability of the Q − V droop (Fig. 2.19):

- the KDroop block represents the Q − V droop characteristic, that computes the reactive power reference from the voltage amplitude
- the block Hinv represents the inverter dynamic, therein approximated with a first-order transfer function between the reactive power reference and the actual output reactive power
- GGrid is the static gain that represents the network, there modeled as an ideal voltage source with certain output impedance
- HDelay block is introduced to model the voltage amplitude measure and filtering and it is approximated with a single Pade’ filter for a constant time delay
- $w_u$ are any disturbances on the output voltage, i.e. induced by load variations

Considering only one DER, if there are no delays in the HDelay block, the system in Fig. 2.19 is always stable because the inverter block Hinv has been modeled as a first-order filter. However, in practice the voltage measure or filtering can make the system unstable [57]. Within this reference, the stability of a single inverter grid first, and then a multiple inverter grid, is studied with a
classical linearization approach. Sensitivity analysis of the eigenvalues of the system are performed to understand how the damping of the system changes according to some parameters, both for a single inverter grid and for a multiple inverter grid. However the work focuses only on inductive grids and it considers only the reactive power loop of the inverters. So the interactions between active and reactive powers that characterize the distribution grids with low X/R ratio cables are not investigated. Moreover, the loads and their dynamic behaviors are not considered, but this approach is interesting and can provide insights on the stability of the Q – V method also for large grids.

![Diagram](image)

**Figure 2.19:** (a) Overview of the investigated inverter system and (b) Simplified block representation of the studied control system, from [57]

### 2.6.2 Frequency support

The national standards for PV DERs connected to the grid have different fixed cutoff frequencies: if the grid frequency rises above these thresholds, the DER has to be disconnected. For example, these frequencies for LV grids are 50.3 Hz in Italy and Denmark and 50.2 Hz in Germany [3, 6]. In an area with a large penetration of PV DERs, an over-frequency event can cause a sudden loss of a large part of generation capacity. In turn, this can cause severe under-frequencies and even rolling blackouts [3]. This has driven the standards to let the PV units to provide frequency support and smoother responses to frequency transients. Different country level
standards now impose that the PV DERs have not to be disconnected suddenly after the grid frequency increases above the threshold, but they have to reduce the power generation gradually. For example, the Italian standard provides the $P-f$ droop curve that is shown in Fig. 2.20 and the German one is similar [3].

Also some papers propose $P-f$ droop characteristics for the PV inverters to support the frequency of the grid. A survey of these works for grid-connected operation of PV is provided by [68]. Usually PV systems are operated at the Maximum Power Point (MPP) to generate all the available power and so only a reduction of the injected power is possible, by moving away from the MPP. If we want to make a PV inverter fully dispatchable, that means having the possibility to increase and decrease the generated power, we have to operate it below the MPP. For example, [69, 70] describe how to realize such controllers also for the implementation of the frequency droop curve and [70] shows how the stability of the grid can be improved with this control technique after severe transients.

![Figure 2.20: P-f droop characteristic for inverter according to Italian standard [6]](image)

**2.6.3 Conclusions of chapter 2**

From this chapter, we can understand that distribution grid scenarios are very complex and varied, because of the presence of different types of device that works simultaneously and with different time scale characteristics. Pure static approaches cannot observe the phenomena generated by their interaction and so dynamic studies of distribution grids are needed, both analytic and of simulation. Among these dynamic approaches, we have noted some common characteristics, as the use of very detailed models for the devices that generates very complicated and less scalable analyses, poor load modeling, even if the load dynamics can play an important role on the grid stability, etc. We also saw that the most used approach for the stability analysis in this context is the small-signal state-space model.

Since the interest of this report activity is on the secondary level dynamics of the grid, it is not necessary to use very detailed approaches, while approximated analyses are preferred because they can address the stability of the grid as a whole. Moreover, this allows obtaining scalable models that are also more handle able and meaningful for large grid studies.

For all these reasons, we are interested in developing simplified models for the devices of the grid and then a scalable small-signal model that links all the device models. Another important point to be considered is the secondary level algorithm of the grid that can influence the voltage stability. We are going to provide an approach to design a secondary level control and to evaluate its dynamics. Among the PV support algorithms of this chapter, we will show an application of our model to the $Q-V$ local droop controller, because it is one of the most considered and it has already been imposed by some country level standards.
3. Grid benchmarks

The increasing diffusion of renewable and distributed energy resources is one of the key challenges of the 21st century. The success of this transition relies on the availability of methods and techniques to assess the economic, technical, and environmental integration of DERs and industry, universities, and research centers all over the world are studying and developing these methods. However, test systems that facilitate the analysis and validation of the developed techniques are still missing.

In the past, the lowest hierarchical levels of the grid were not object of particular research interests since they were dominated only by passive users. However, during the last years, as DG is increasing more and more DERs are usually connected to the end-user side, at the LV distribution network. The recent interest in these parts of the grid has pushed the researchers to develop some standard benchmarks to study the distribution grid. However, this interest is only recent and these benchmarks are not so detailed.

In this chapter, we illustrate some grids used to study issues and problems at the LV and MV levels of the grid; first, focusing on the IEEE standard benchmarks and then to the CIGRE test-cases. Furthermore, a distribution grid model, got form one partner of IDE4L Project, is described. This grid will be used as standard benchmark in the following chapters.

3.1 IEEE benchmarks

Several grid benchmarks that have been proposed so far regard large power systems, considering HV levels and/or transmission side of the grid, while distribution side, and in particular LV grid descriptions, usually lacks. For example, the IEEE Standard 399-1997 [71] provides some MV benchmarks and some descriptions to address power system problems, as for instance load flow analysis, short-circuit study, stability analysis, etc. Regarding the stability analysis section, this reference focuses on the ability of the power system to operate in presence of two or more synchronous machines. The stability is studied as steady-state stability, i.e. the ability of a power system to maintain synchronism between machines within the system following relatively slow load changes, as the transient stability that is the ability of the system to remain in synchronism under transient conditions, as faults, switching operations, etc.

![LV benchmark with DG obtained from IEEE Standard 399-1997](image)

Figure 3.1: LV benchmark with DG obtained from IEEE Standard 399-1997 [71] that is used in [73]
So these test-cases usually do not contain renewable DERs. Nevertheless, some works that focus on distribution grid aspects start from [71] to build their own grid test-case. For example, [72] takes the system configuration and parameters from the IEEE Standard 399-1997, but some modifications are done to study the autonomous microgrid operation. Also in [73], the used test-case is obtained from [71], but some adjustments are made to study a LV grid with distributed generators (this grid is in Fig. 3.1). Standard MV distribution test feeders are provided by the IEEE Energy & Power Society [74]. However these benchmarks do not include DERs and they are always MV grids and not LV ones. Some examples are the 24, 9 kV distribution grid in Fig. 3.2 (IEEE 34 test feeder) and the 4, 16 kV in Fig. 3.3 (IEEE 13 test feeder). In the last benchmark, there is also an in-line transformer that reduces the voltage to 480 V for a short section of the feeder. Other MV benchmarks are presented in [74], including a 123 nodes test-feeder, 37 node test-feeder and a 4 node test-feeder, however all these benchmarks do not consider DG. Lots of works and papers involve these benchmarks as starting point for their analyses and add particular devices that have to be investigated: among the most used test-cases, there are the IEEE Standard 399-1997 and IEEE 34 test feeder.

A distribution grid with MV and LV sides is described in [75] and here reported in Fig. 3.4. This grid includes DERs and loads and it is more detailed and complete for our purposes than the previous ones. The greatest part of papers that focus on distribution grid issues or related topics usually develops its own benchmark, sometimes starting from the grids that have been shown in this section. In the next section, another set of benchmarks for MV and LV applications is presented.

Figure 3.2: IEEE 34 test feeder from [74]
IDE4L is a project co-funded by the European Commission

Figure 3.3: IEEE 13 test feeder from [74]
IDE4L is a project co-funded by the European Commission
Figure 3.4: One-line diagram of CERTS microgrid test-bed [75]
3.2 CIGRE benchmarks

The technical report [76] proposes some benchmark grids for HV, MV and LV levels, all of them both for American and European grid applications. In this section, we summarize the interesting contents of such report for the application to the IDE4L Project. Thus, we will focus only on the distribution grid, i.e. the MV and LV levels, for the European applications. Fig. 3.5 shows a schematic representation of the contents of [76] and the parts that we consider in this section.

3.2.1 Benchmark applications

The grid benchmarks are useful for the analysis, the design, and the validation of methods for the network integration of renewable and distributed energy resources. As Fig. 3.5 shows, these benchmarks can be classified in resource-side benchmarks, that focus on the operation of a single device (e.g. one particular DER), and network benchmarks, that focus on the integration of different units.

In IDE4L Project, we are interested in stability issues of the distribution grid, in particular due to the integration of several DERs. This integration could cause poor stability problems, in terms of small-signal angle stability, transient stability and voltage stability [76]. These considerations drive us to analyze only the network benchmarks for DER integration aspects, rather than considering the operation of a single unit. Tab. 3.1 provides some specific examples and application contexts for these benchmarks.

3.2.2 Resource-side benchmark

These guidelines are provided by [76] to analyze, study and develop specific resource-side topologies and control strategies. Analyzing only one unit, it is possible to consider an equivalent connection to the upper level grid, as a voltage source behind a series impedance, as in Fig. 3.6. This equivalence can be determined by the voltage level $V_G$, the short-circuit power $SSC$ and the impedance $Z_L$ of the grid-connection, as in Tab. 3.2. No variations of frequency or amplitude or dynamics are considered in this test-case.
### Table 3.1: Application matrix for network benchmarks [76]

<table>
<thead>
<tr>
<th>Context</th>
<th>Issue</th>
<th>Example study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operation and control</td>
<td>Demand side response</td>
<td>Quantifying the benefit of demand side response</td>
</tr>
<tr>
<td></td>
<td>Energy management</td>
<td>Managing storage for wind energy conversion</td>
</tr>
<tr>
<td></td>
<td>Emissions displacement</td>
<td>Value of renewable energy sources for emissions displacement</td>
</tr>
<tr>
<td></td>
<td>Frequency control</td>
<td>Keeping the power balance in a microgrid</td>
</tr>
<tr>
<td></td>
<td>Optimal power flow Security</td>
<td>Optimizing losses in the presence of DG</td>
</tr>
<tr>
<td></td>
<td>Vehicle-to-grid (V2G)</td>
<td>Self-healing network algorithms</td>
</tr>
<tr>
<td></td>
<td>Voltage control</td>
<td>Designing V2G controls</td>
</tr>
<tr>
<td></td>
<td>Unit commitment</td>
<td>Impact of DG on control of distribution and transmission</td>
</tr>
</tbody>
</table>

| Planning and design | DER siting | DER siting for congestion mitigation |
| | Distribution reinforcement | Ratings in the presence of DER |
| | Investment decision | Quantify value of DG versus line investment |
| | Network extension | Transmission planning for wind interconnections |

| Power quality | Ferro resonance | Risk of Ferro resonance in transformers connecting DG |
| | Flicker Harmonics | Effect of distributed wind on flicker |
| | Motor starting | Effect of DG on harmonics in networks |
| | Service interruption | Providing starting currents with DER |
| | Unbalance | System average interruption duration and frequency indices |
| | Voltage profile | Impact of single-phase DG connections |

| Protection | Coordination of devices | Coordinating devices such as reclosers |
| | Fault current | Desensitization of relays through DG |
| | Fault voltage | Voltage support through DG |
| | Insulation coordination | Comparing different strategies Testing for overvoltages |
| | Relay tripping | Impact of DG on unwanted tripping |

| Stability | Islanding | Enhancing stability with controlled islanding |
| | Low voltage ride through Small-signal angle stability | Impact of LVRT profile on system stability |
| | Stabilizer design | Angle stability in networks with off-shore wind farms |
| | Transient stability | Testing stabilizers for DG in real-time hardware-in-the-loop |
| | | Response of DER subject to large disturbances |

### Table 3.2: Parameters of European grid equivalent of resource-side benchmark [76]

<table>
<thead>
<tr>
<th>Voltage level</th>
<th>Grid voltage VG [ kV, rms LL ]</th>
<th>Short circuit power SSC [ MVA ]</th>
<th>R/X Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low voltage</td>
<td>0,4</td>
<td>1 to 10</td>
<td>0,70 to 11,0</td>
</tr>
<tr>
<td>Medium voltage</td>
<td>20</td>
<td>100 to 1000</td>
<td>0,40 to 2,00</td>
</tr>
<tr>
<td>High voltage</td>
<td>220</td>
<td>5000 to 20000</td>
<td>0,07 to 0,60</td>
</tr>
</tbody>
</table>
3.2.3 MV distribution network benchmark

A MV benchmark is described by [76] in terms of nominal parameters (20 kV and 50 Hz) and structure (topology, three-phase, meshed or radial, etc.). All the data of cables (types, lengths, impedances, etc.) and of the transformers is given, together with the grounding connections and the load data. Unbalance is not explicitly included, but it can be introduced if desired. However in this grid, no DERs are explicitly considered and only the nominal values of the loads are given without entering in any details (no load descriptions are provided). The report [76] provides flexibility and suggestions about how the benchmark can be modified, i.e. how to change the parameters of the loads, cables, etc. Further suggestions for these modifications are given in [77].

3.2.4 LV distribution network benchmark

A LV benchmark is described by [76] and reported here in Fig. 3.9 and some of its parameters are in Tab. 3.4. This grid is described in terms of nominal parameters, structure, line types (lengths and impedances). The grid is split in three parts with different characteristics: residential, industrial and commercial grids. Other details are provided concerning ground connections, transformers, loads and also symmetry of the grid (there are single-phase consumers). Flexibility and suggestions are given by [76] to modify the benchmark, i.e. how to change the parameters of loads, cables, etc. However in this grid, no DERs are explicitly considered and only the nominal values of the loads are given without entering in any details (no load descriptions are provided). An example of modification of the benchmark in Fig. 3.9 is also provided by [76], together with some simulation results. This grid is shown in Fig. 3.10. This test-bed is used by [76] to show an islanded operation of some DERs; anyway no dynamic descriptions of the loads are provided and only static considerations are drawn.
3.3 A2A grid benchmark

The partner A2A of IDE4L Project has provided data about a real distribution grid, divided in MV and LV levels. Since all the DERs are located at the LV side, we focus our studies on this part of the grid. This grid consists of 2 LV feeders, 19 monitored customers and 49 customers for state estimation. This grid has a nominal frequency of $\omega_0 = 2\pi 50$ Hz and nominal line to neutral peak voltage of $E_0 = 230v2 \sqrt{2}$ V, for the LV side. Observe that all the DERs are PV panels and are single-phase. Also almost all the customers have single-phase connections and only rated powers for the loads and the generation sides are provided.
3.4 Conclusions of chapter 3

This chapter has shown that the grid benchmarks for distribution networks are often not fully described; they usually provide only the topology and the cable descriptions. Concerning the devices, loads and generators, usually very few details are provided.

The A2A grid description includes: topology, cables, connections of the loads and DERs are all provided. Also the types of the DERs and the rated powers of loads and generators are given. In the next chapter, a mathematical tool to address the small-signal stability of a distribution grid is presented and then it is applied to study the A2A grid in chapter 5. We need only to choose the modeling of the PV units and of the loads, but for them we can exploit what we saw in Sec. 2.4.

4. Small signal stability analysis

In this chapter a small-signal approach is presented to study the dynamic behavior of a distribution grid that has a large penetration of DG, in particular referring to the A2A grid, described in Sec. 3.3. A linear model for the electric grid is first obtained and then a non-linear model for the users (loads and generators) is described. From this model, through a linearization process, the small-signal model of all the users of the grid is written and then linked to the model of the electric grid. This result enables the dynamic study of the network and also the design of a secondary level controller for DERs. So, an application of the local Q - V droop regulator for DERs (Sec. 2.6.1) is described: this is modeled in the same manner to study how the dynamic characteristics of the grid change when several DERs have voltage support capabilities.

4.1 Open-loop small-signal model of the grid

The following analysis exploits the space vectors defined in the rotating reference frame (dq0 or Park transformation [22]) to describe the currents and the voltages of the grid. This choice enables to represent with no approximations all the currents and the voltages of the grid, as shown later. So all the sinusoidal quantities, even in the presence of variations of amplitude and frequency, can correctly be described. For symmetrical three-phase systems, the dq transformation is well defined, but this description can be applied also to single-phase systems, considering them as a part of a fictitious balanced three-phase system. That is, the single-phase system is one of the three phases of a balanced three-phase system and the other two phases derive from the first one after a phase shift of 120° and 240°, respectively.

4.2 Dynamic network model

Consider a single-phase resistive-inductive electric line, as in Fig. 4.1. If $\xi(t) \in R$ is the instantaneous inductor current, the equation that relates voltages and current is:

$$L \frac{d\xi_k(t)}{dt} = u_i(t) - u_j(t) - R\xi_k(t) \quad (4.1)$$

where $u_i(t) \in R$ and $u_j(t) \in R$ are the instantaneous voltages respectively at the beginning and at the end of the line. From now, the time dependencies is dropped to simplify the notation.

Figure 4.1: RL electric line of the grid
If the RL line belongs to a symmetric three-phase system or if it is a single-phase system, we can express the Eq. (4.1) with the space vectors defined in the stationary reference frame $\alpha - \beta$ [22].

$$L \frac{d\vec{x}^{\alpha\beta}}{dt} = \vec{u}^{\alpha\beta} - \vec{u}^{\alpha\beta} - R \vec{x}^{\alpha\beta} \quad \text{in} \quad \alpha - \beta$$

(4.2)

where the bar $\bar{\cdot}$ symbol refers to a complex quantity that is called space vector. Applying the Park transformation [22] that expresses a generic space vector $\vec{g}^{\alpha\beta}$ described in the stationary reference frame in a space vector $\vec{g}^{dq}$ that is described in the rotating reference frame, here indicated with $d-q$ frame, we get:

$$\vec{g}^{\alpha\beta} = \vec{g}^{dq} e^{j\omega t}$$

(4.3)

Here, this transformation is performed on a reference frame that rotates with constant angular frequency which is equal to the nominal angular frequency of the system $\omega_0 = 2\pi 50 \text{ rad/s}$ and with a null initial angle. With $j$ we indicate the imaginary unit of the complex numbers. Transforming Eq. (4.2), using Eq. (4.3), it comes that:

$$L \frac{d\vec{x}^{dq}}{dt} = \vec{u}^{dq} - \vec{u}^{dq} - (R + j\omega L) \xi^{dq} \quad \text{in} \quad d-q$$

(4.4)

Now we write all the Kirchhoff’s voltage laws (LKV) and Kirchhoff’s currents laws (LKI) for the grid using a compact form, thanks to the matrix notation. In order to do this, a graph theory approach is exploited as in [78], assuming that the grid has $N_n$ nodes and $N_e$ edges or branches. All the LKV for the grid are:

$$L_k \frac{d\xi}{dt} = \vec{u}_i - \vec{u}_j - (R_k + j\omega L_k) \xi_{k} \quad k = 1, \ldots, N_e$$

(4.5)

where all the space vectors are defined in the rotating reference frame (the $dq$ superscripts have been dropped for sake of simplicity) and $k$ index identifies the edge of the graph with resistance $R_k$ and inductance $L_k$. To write all the Eq. (4.5) equations for all the edges, the incidence matrix $A$ is defined as in [78]:

$$A \in \{0, \pm 1\}^{N_e \times N_n} : a_{vr} = \begin{cases} -1 & \text{if the edge e starts form the node v} \\ 1 & \text{if the edge e arrives at the node v} \\ 0 & \text{otherwise} \end{cases}$$

(4.6)

Let us define the two diagonal matrices $L$ and $R$ as:

$$L = \text{diag} (L_1, L_2, \ldots, L_{N_e}) \in \mathbb{R}^{N_e \times N_e} \quad R = \text{diag} (R_1, R_2, \ldots, R_{N_e}) \in \mathbb{R}^{N_e \times N_e}$$

(4.7)

where $\text{diag} (\cdot)$ denotes a diagonal matrix having the entries of the vector as diagonal elements, and the two vectors $\xi$ and $U$ as:
where the T operation refers to the matrix transposition. Note that \( \xi \) and \( U \) are complex vectors that contain the space vectors of all the edge currents and node voltages respectively. Considering all the previous definitions, the LKV of Eq. (4.5) can be written in this compact form:

\[
\frac{d\xi}{dt} = -AU - (R + j\omega L) \xi
\]

(4.9)

and since \( L \) is diagonal, thus invertible:

\[
\frac{d\xi}{dt} = -L^{-1}AU - (L^{-1}R + j\omega L^{-1}I_{Ne}) \xi
\]

(4.10)

where \( I_{Ne} \) is the identity matrix of dimensions \( N_e \times N_e \). Indicate with \( \vec{\imath}_k \) the space vector (in the dq reference frame) of the current that is injected at the node \( k \), for example by a load or a DER. All the LKI can be written by summing all the currents that flow toward each node, but before let us define the vector \( J \) as:

\[
J = \begin{bmatrix} \vec{\imath}_1 & \vec{\imath}_2 & \cdots & \vec{\imath}_N \end{bmatrix}^T \in \mathbb{C}^{N_e \times 1}
\]

(4.11)

as the vector of all the space vectors that represent the node currents. Thus, a compact way to write all the LKI is:

\[
A^T \xi + J = 0
\]

(4.12)

Merging the LKV and LKI equations, respectively Eq. (4.10) and Eq. (4.12), the model of the grid is obtained:

\[
\begin{cases}
\frac{d\xi}{dt} &= - (L^{-1}R + j\omega L^{-1}I_{Ne}) \xi - L^{-1}AU \\
J &= -A^T \xi
\end{cases}
\]

(4.13)

Note that the model (4.13) is a linear model that is described as state-space model: \( \xi \) is the state vector, \( U \) the input vector and \( J \) the output vector. This model has been derived for the currents and voltages of the grid, but the same model is still applicable in a small-signal context, that is the same model describes also the deviations of voltages and currents from their respective steady-state operating point. All the matrices and vectors in Eq. (4.13) are complex and, to exploit the well-known results of the state-space theory, we need to write such model with real matrices and vectors:

\[
\begin{bmatrix}
\text{Re}(\xi) \\
\text{Im}(\xi)
\end{bmatrix}
\begin{bmatrix}
\frac{d}{dt} \\
\frac{d}{dt}
\end{bmatrix}
= 
\begin{bmatrix}
-\text{Re}(L) & -\text{Im}(L) \\
-\text{Im}(L) & \text{Re}(L)
\end{bmatrix}
\begin{bmatrix}
\text{Re}(\xi) \\
\text{Im}(\xi)
\end{bmatrix}
+ 
\begin{bmatrix}
L^{-1}A & 0 \\
0 & L^{-1}A
\end{bmatrix}
\begin{bmatrix}
\text{Re}(U) \\
\text{Im}(U)
\end{bmatrix}
+ 
\begin{bmatrix}
\text{Re}(L) \\
\text{Im}(L)
\end{bmatrix}
\begin{bmatrix}
\text{Re}(J) \\
\text{Im}(J)
\end{bmatrix}
\]

(4.14)
the final dynamic model for the grid results:

\[
\frac{dx}{dt} = A_x x + B_x u, \\
J_s = C_x x
\] (4.16)

The singular perturbation theory describes how to approximate a differential equation system that has a set of fast dynamics and a set of slow dynamics [79]. Under appropriate hypotheses, we can replace the set of faster equations, i.e., the equations that describe the fast dynamic variables, with a set of algebraic equations. With this approximation, the model loses the information about fast modes, assuming the state variables associated to these modes to be always in a steady-state condition. In this way, the order of the differential equation system can be reduced, as well as the complexity of the system itself. In a distribution grid, there are different time-scale dynamics and, among these, the dynamics of the electric cables can be considered as fast [21]. This means that the differential equation system for the network dynamics, for example Eq. (4.13), usually describes the fastest dynamics within the differential equation system for the whole power system, for example Eq. (2.2). In this context, we can approximate the dynamic model of the grid that has been presented in Sec. 4.1.1 with a static model for the phasors at the nominal angular frequency \(\omega_0\). With other words, the dynamic model (4.13) can be approximated with a full algebraic model by setting the time derivative of \(\xi\) equal to zero:

\[
AU + Z\xi = 0 \quad \Rightarrow \quad \xi = -Z^{-1}AU
\] (4.17)

where \(Z = R + j\omega_0 L \in C^{N_x \times N_x}\) is diagonal and so invertible. Substituting the last equation in the second one of Eq. (4.13), we get the static model of the grid:

\[
J = (A^T Z^{-1} A) U
\] (4.18)

Note that this model corresponds to the classic formulation with the admittance matrix of Eq. (2.1). As with (4.14), we write the model (4.18) with real matrices and vectors:

\[
\begin{bmatrix}
\text{Re}(J) \\
\text{Im}(J)
\end{bmatrix} =
\begin{bmatrix}
\text{Re}(Z^{-1}AU) & -\text{Im}(Z^{-1}AU) \\
\text{Im}(Z^{-1}AU) & \text{Re}(Z^{-1}AU)
\end{bmatrix}
\begin{bmatrix}
\text{Re}(U) \\
\text{Im}(U)
\end{bmatrix}
\] (4.19)

and defining:

\[
H_g =
\begin{bmatrix}
\text{Re}(Z^{-1}AU) & -\text{Im}(Z^{-1}AU) \\
\text{Im}(Z^{-1}AU) & \text{Re}(Z^{-1}AU)
\end{bmatrix}
\] (4.20)

it follows that:
with the same definitions for \( J \) and \( U \), shown in (4.15b).

### 4.2.2 Model of the PCC

Among all the nodes of the grid, one is special and it is where there is the connection to the higher voltage level, for example the MV/LV transformer location. This node has a special model, different from all the other nodes, and it is described in this section.

We have decided to model it as an ideal voltage generator with certain output impedance, as done by several works on grid-connected operation for DERs, which we reported in chapter 2 and also in Sec. 3.2. This modeling choice neglects the dynamics of the frequency that so is assumed to be constant. The reason of this relies on the limited rated power that is usually installed in LV grids, compared to the one of the rest of the power system (also in the grid that we are going to consider). As we described in Sec. 2.5.2, transients as load variations or PV intermittency within the LV grid have a negligible influence on the frequency. On the other hand, the impedance of the PCC is accounted by the edge that departs from this node. Assuming that the PCC is at the node 1, its model can be written as:

\[
\bar{u}_{PCC} = \bar{u}_1 = \bar{E}_0 \in \mathbb{R} \quad \forall \quad \bar{i}_1 \in \mathbb{C}
\]  

(4.22)

where the space vectors have been used to represent the PCC voltage \( u^*1 \) and its injected current \( i^*1 \). Since the \( d-q \) space vector is constant, the PCC voltage is a sinusoidal waveform with angular frequency equal to the nominal one \( \omega_0 \) and with amplitude equal to the nominal line to neutral voltage \( E_0 = 230*\sqrt{2} \). Splitting Eq. (4.22) in its real \( u_{d1} \) and imaginary \( u_{q1} \) components \( (\bar{u}_1 = u_{d1} + ju_{q1}) \) and then obtaining a small-signal model, it follows:

\[
\begin{cases}
\hat{u}_{d1} = 0 \\
\hat{u}_{q1} = 0
\end{cases}
\]  

(4.23)

where the hat symbol (\( \hat{\} \)) refers to a small-signal perturbation of that variable around the steady-state solution where the small-signal analysis is performed. Those components are both null, because the space vector \( u^*1 \) is constant and so its real and imaginary components. Later, the \( d \) and \( q \) subscripts will indicate respectively the real and the imaginary part of a space vector or of a complex quantity.

### 4.2.3 Model of the nodes

The same model for each node of the grid is considered to make scalable and automatable the building of the final model. Each node consists of the parallel connection of (see Fig. 4.2):

- a series of a resistance and an inductance, that can roughly model a motor or other loads as seen in Sec. 2.4.2;
- a capacitor, that can represent a real capacitor connected to the grid or the shunt capacitance of the cables;
- a constant power generator with a first-order dynamic of the current, that approximates the DER;
- a constant power load with a first-order dynamic of the current, that can model dynamic loads (for example active loads).

By setting different values for these elements, we can describe different types of node. More details about the generator modeling are provided later while describing its mathematical model.
As so far, we describe the RLC load with space vectors that are defined in the $d-q$ frame. The model that relates the instantaneous voltage and currents is (see Fig. 4.2):

$$
\begin{align*}
C \frac{di(t)}{dt} &= i_g(t) - i_r(t) - i(t) \\
L \frac{di_r(t)}{dt} &= u(t) - Ri_r(t)
\end{align*}
$$

Transforming this system, before in the stationary reference frame $\alpha-\beta$ and then in the rotating reference frame $d-q$, we get:

$$
\begin{align*}
C \frac{di_g}{dt} &= \tilde{i}_g - \tilde{i}_r - \tilde{i}_l - j\omega_0C\tilde{u} \\
L \frac{di_r}{dt} &= \tilde{u} - (R + j\omega_0L)\tilde{i}_r
\end{align*}
$$

where the time dependencies ($t$) and the $dq$ symbols have been dropped for sake of simplicity. Focusing on grid-connected inverters for PV applications, they usually are controlled as current sources that inject all the available active power at unity PF [4]. Now, we want also to consider inverters that can inject power at non-unity PF for grid supporting functions, as in chapter 1 and Sec. 2.6. For this reason, here we describe the PV inverter as current source with a first-order approximation on the active and reactive power tracking capabilities: the inverter is controlled with a complex power reference (active and reactive). While the active power dynamic can derive from the speed convergence of the MPPT algorithm [80], the reactive power dynamic can come from the presence of a closed regulation loop that adjusts properly the generated current. The same choice is done by [57], as we saw in Sec. 2.6.1. The first-order model for the constant power source and load has also been used in [8]:

$$
\begin{align*}
\tau_g \frac{d\tilde{i}_g}{dt} &= -\tilde{i}_g + 2\left(\frac{\tilde{s}_g}{\tilde{u}}\right)^* \\
\tau_l \frac{d\tilde{i}_l}{dt} &= -\tilde{i}_l + 2\left(\frac{\tilde{s}_l}{\tilde{u}}\right)^*
\end{align*}
$$

where $s_g \in \mathbb{C}$ and $s_l \in \mathbb{C}$ are the complex power references respectively for the PV generator and the load. So for the load, we consider the parallel of a RLC load, that is modeled with the exact dynamics of the capacitor and the inductance, rather than with a phasorial representation and a dynamic model that is the same used for the DER. Consider that by setting a fast time-constant, the load behaves like a constant power load and, by setting a slow time constant, it can behave like a constant current load. So this modeling choice can include and describe all the major load models that we saw in Sec.2.4.2. Considering, generically, the
last two terms in the two equations of (4.26) as:

\[ i_{ref} = 2 \left( \frac{\theta}{\Omega} \right)^* \]  

(4.27)

and expressing \( \bar{u} = u_d + j u_q \) and \( i_{ref} = i_d + j i_q \) with the real and imaginary parts, we can write also Eq. (4.27) with its real and imaginary parts:

\[
\begin{align*}
  p &= \text{Re}(\hat{s}) = \frac{1}{2} (u_d i_d + u_q i_q) \\
  q &= \text{Im}(\hat{s}) = \frac{1}{2} (u_d i_d - u_q i_q)
\end{align*}
\]  

(4.28)

Eq. (4.28) is non-linear and since we want to develop a small-signal state-space model we have to linearize it:

\[
\begin{bmatrix}
  \dot{p} \\
  \dot{q}
\end{bmatrix} = \begin{bmatrix}
  \frac{1}{2} U_d & U_q \\
  U_q & -U_d
\end{bmatrix} \begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} + \begin{bmatrix}
  \frac{1}{2} I_d & I_q \\
  -I_q & I_d
\end{bmatrix} \begin{bmatrix}
  u_d \\
  u_q
\end{bmatrix}
\]  

(4.29)

where the capital letter indicates the value of the corresponding variable evaluated at the operating point where the linearization is performed. Since \( \det A = -U_d / 2 - U_q / 2 \) is equal to the opposite of the square of the voltage amplitude of the node and in general it is not zero for all the nodes, the matrix \( A \) can be inverted:

\[
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} = A^{-1} \begin{bmatrix}
  \dot{p} \\
  \dot{q}
\end{bmatrix} - A^{-1} B \begin{bmatrix}
  u_d \\
  u_q
\end{bmatrix}
\]  

(4.30)

Now Eq. (4.26) can be linearized as it has been shown with Eq. (4.30). Consider that the matrix \( A \) is the same for the constant power load and the generator since their voltage is the same (same node), but \( B \) is different. For this reason, \( B_g \) indicates the \( B \) matrix of the generator and \( B_l \) indicates the \( B \) matrix of the load in Eq. (4.30). It follows that:

\[
\begin{align*}
  \frac{d}{dt} \begin{bmatrix}
    i_{rd} \\
    i_{rq}
  \end{bmatrix} &= -A^{-1} B_g \begin{bmatrix}
    i_{rd} \\
    i_{rq}
  \end{bmatrix} - \frac{1}{C} \begin{bmatrix}
    i_{rd} \\
    i_{rq}
  \end{bmatrix} + \frac{A^{-1}}{L} \begin{bmatrix}
    u_d \\
    u_q
  \end{bmatrix} \\
  \frac{d}{dt} \begin{bmatrix}
    i_{ld} \\
    i_{lq}
  \end{bmatrix} &= -A^{-1} B_l \begin{bmatrix}
    i_{ld} \\
    i_{lq}
  \end{bmatrix} - \frac{1}{C} \begin{bmatrix}
    i_{ld} \\
    i_{lq}
  \end{bmatrix} + \frac{A^{-1}}{L} \begin{bmatrix}
    u_d \\
    u_q
  \end{bmatrix}
\end{align*}
\]  

(4.31)

Let us split the relations (4.25) in their real and imaginary parts to build at the end a state-space model with matrices with real elements. Then, writing the resulting model (that is linear) for the small-signal variations, it follows:

\[
\begin{align*}
  \frac{d}{dt} \begin{bmatrix}
    i_d \\
    i_q
  \end{bmatrix} &= \begin{bmatrix}
    0 & \omega_0 \\
    -\omega_0 & 0
  \end{bmatrix} \begin{bmatrix}
    i_d \\
    i_q
  \end{bmatrix} - \frac{1}{C} \begin{bmatrix}
    i_{rd} \\
    i_{rq}
  \end{bmatrix} + \frac{1}{C} \begin{bmatrix}
    i_{pd} \\
    i_{pq}
  \end{bmatrix} - \frac{1}{C} \begin{bmatrix}
    i_{ld} \\
    i_{lq}
  \end{bmatrix} \\
  \frac{d}{dt} \begin{bmatrix}
    i_{rd} \\
    i_{rq}
  \end{bmatrix} &= \frac{1}{L} \begin{bmatrix}
    i_d \\
    i_q
  \end{bmatrix} \begin{bmatrix}
    -R/L & \omega_0 \\
    -\omega_0 & -R/L
  \end{bmatrix} \begin{bmatrix}
    i_d \\
    i_q
  \end{bmatrix} + \begin{bmatrix}
    i_{rd} \\
    i_{rq}
  \end{bmatrix}
\end{align*}
\]  

(4.32)

To write the state-space model of a single node (Fig. 4.2), define the following vectors:
and the following matrices:

\[
\begin{bmatrix}
0 & \omega b & -1/C_i & 0 & 1/C_i & 0 & -1/C_i & 0 \\
-\omega b & 0 & 0 & -1/C_i & 0 & 1/C_i & 0 & -1/C_i \\
1/L_i & 0 & -R_i/L_i & \omega b & 0 & 0 & 0 & 0 \\
0 & 1/L_i & -\omega b & -R_i/L_i & \omega b & 0 & 0 & 0 \\
-\Lambda^{-1}b_{pi} & \nu_p & 0 & -1/\tau_{pi} & 0 & 0 & 0 & 0 \\
-\Lambda^{-1}b_{ni} & \nu_n & 0 & 0 & -1/\tau_{pi} & 0 & 0 & 0
\end{bmatrix} \in \mathbb{R}^{8 \times 8}
\]

(4.34a)

\[
B_{ni} = \begin{bmatrix}
-1/C_i & 0 \\
0 & -1/C_i \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix} \in \mathbb{R}^{8 \times 2} \quad D_{ni} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & \frac{\Delta^{-1}}{\nu_p} \\
0 & 0 \\
0 & \frac{\Delta^{-1}}{\nu_n}
\end{bmatrix} \in \mathbb{R}^{8 \times 4}
\]

(4.34b)

Observe that in the definitions (4.33) and (4.34), the subscript i has been added to each quantity to refer to the node number, that is i = 2, . . . , N_n. Node 1 does not have to be considered because it is the PCC and it is modeled as in Sec. 4.1.3. In this way, the small-signal models of the nodes of the grid result.

\[
\frac{d}{dt} \dot{z}_i = A_{ni}\dot{z}_i + B_{ni}J_i + D_{ni}S_i \quad i = 2, \ldots, N_n
\]

(4.35)

Since, the grid models, both (4.16) and (4.21), receive as input the voltages of all the nodes, we keep as outputs of the node model the node voltages u’d and u’q:

\[
U_i = C_{ni}\dot{z}_i \quad \text{where} \quad C_{ni} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad \text{and} \quad U_i = \begin{bmatrix}
\theta_{ih} \\
\theta_{iq}
\end{bmatrix}
\]

(4.36)

and C_{ni} \in \mathbb{R}^{2 \times 8} and U_i \in \mathbb{R}^{2 \times 1}.

\[
\begin{aligned}
\frac{d}{dt} z &= A_nz + B_nJ + D_nS \\
U &= C_nz
\end{aligned}
\]

(4.37)

Merging the models of all the nodes, that are in Eq. (4.35) and (4.36), we get:
Observe that two null columns have been added to the matrix $B_n$ because the vector $J$ has as first elements the PCC currents, as we can see in the definition (4.38a). Similarly, two null rows have been added to the matrix $C_n$ to account the PCC voltage components that are in the vector $U$, see definition (4.38b).

### 4.2.4 Overall small-signal model

Once we get the model of all nodes (4.37) and model of the grid, (4.16) or (4.21), we can obtain the model of the complete system. However, we have to observe that the arrangements of vectors $J$ and $U$, in Eq. (4.37) and in Eq. (4.16) or (4.21) are different, and for this reason some row and column permutations are needed in the $B_n$ and $C_n$ matrices before joining the two models. These trivial details will be skipped for briefness reasons. After doing these arrangements, the model (4.37) and (4.16) can be linked to obtain the overall small-signal model:
On the other hand, merging Eq. (4.37) and (4.21), we get:

\[
\frac{d}{dt} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_n & B_n C_n \\ B_n C_n & A_g \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \begin{bmatrix} D_n \\ 0 \end{bmatrix} S
\]

(4.41)

Once the state-space model, as (4.40) or (4.41), has been obtained we can use the standard tools of state-space feedback control to design a controller, either local or centralized, for the DERs. Furthermore, the evaluation of the eigenvalues of the matrix $A_{tot}$ in Eq. (4.40) or Eq. (4.41) allows us to study the dynamics of distribution grid, under the approximation hypotheses that have been done in this section.

**4.3 Local Q-V droop control application**

The small-signal model presented so far is now extended to study the Q-V droop control according to [6] and as described in Sec. 2.6.1. Since in the model of the process, Eq. (4.40) or Eq. (4.41), the state $z$ contains the voltage deviations of the real and imaginary components ($d - q$ components), we have to relate them to the voltage amplitude, that is the input for such local controller. Consider an inverter that is connected at the node $i$ with the voltage $u_i = u_{di} + ju_{qi}$. The voltage amplitude $U_i$ is related to the real and imaginary components as:

\[
|\tilde{u}_i| = U_i = \sqrt{u_{di}^2 + u_{qi}^2}
\]

(4.42)

and linearizing this equation:

\[
\tilde{U}_i = \frac{U_{di}}{\sqrt{U_{di}^2 + U_{qi}^2}} \dot{u}_{di} + \frac{U_{qi}}{\sqrt{U_{di}^2 + U_{qi}^2}} \dot{u}_{qi}
\]

(4.43)

where $U_{di}$ and $U_{qi}$ refer to the steady-state $d$ and $q$ components of the node voltage where the linearization is performed, and $\dot{u}_{di}$ and $\dot{u}_{qi}$ refer to their small-signal variations. $U_i$ and $\tilde{U}_i$ are respectively the voltage amplitude steady-state solution and its small-signal variation. Writing all the Eq. (4.43) for all the nodes of the grid, we can get all the variations of the voltage amplitudes according to the variations of the $d$ and $q$ components of all the nodes. Remembering that these last quantities are included in the state vector of the grid models (in particular in $z$), for example Eq. (4.40), we can write:

\[
\hat{U} = T_{ctr} \hat{z} \quad \begin{pmatrix} u \\ \dot{u} \end{pmatrix} \Rightarrow \hat{U} = T_{ctr} \hat{z}
\]

(4.44)

where the subdivision of the two matrices has been done accordingly. A similar relation can be obtained also from the model in Eq. (4.41): it is equal to the second one in (4.44). To apply the Q-V droop characteristic, the inverter controller has to measure the amplitude voltage at its terminals. In order to describe this measure in the mathematical model, this work considers a first-order filter between the actual voltage amplitude $U_i$ and the measured voltage amplitude $U_{m}$. With a differential equation, it is:
\[
\frac{dU_m}{dt} = -\omega_{ri}U_m + \omega_{ri}U_i \quad i = 1, 2, \ldots, N_n
\]  

(4.45)

where the \(m\) subscript refers to the measure. A different modeling choice for the voltage measure is done by [57] (Sec. 2.6.1), which considers a constant time delay, described with a Pade’ approximation. Defining the two vectors:

\[
U_m = [U_{m1}, U_{m2}, \ldots, U_{mN_n}]^T \in \mathbb{R}^{N_n 	imes 1} \quad \hat{U} = [\hat{U}_1, \hat{U}_2, \ldots, \hat{U}_{N_n}]^T \in \mathbb{R}^{N_n 	imes 1}
\]  

(4.46)

and the diagonal matrix \(\Omega_r = \text{diag} (\omega_{r1}, \omega_{r2}, \ldots, \omega_{rN_n})\), we can write Eq. (4.45) for all the nodes of the grid, getting: Once the voltage amplitude measure is described, the reactive power reference of the inverter is changed according to the \(Q-V\) droop characteristic [6]. Since we are interested in a small-signal model, we can linearize this non-linear characteristic as:

\[
\frac{d\hat{U}_m}{dt} = -\Omega_r \hat{U}_m + \Omega_r \hat{U}
\]  

(4.47)

Once the voltage amplitude measure is described, the reactive power reference of the inverter is changed according to the \(Q-V\) droop characteristic [6]. Since we are interested in a small-signal model, we can linearize this non-linear characteristic as:

\[
\hat{q}_{ref} = -k_v \hat{U}_m
\]  

(4.48)

where \(k_v\) is the slope of the curve in the particular operating point where the linearization is performed. Observe that \(k_v\) can be a positive constant if the voltage of the inverter is within the limit 0, 9+0, 92 p.u. or 1, 08 ÷ 1, 1 p.u., otherwise it is zero. Including Eq. (4.48) for all the inverters of the grid, we can write the vector \(S\) of (4.38b) as a function of the vector \(U_m\):

\[
S = K_{ctr} \hat{U}_m \quad \text{where} \quad K_{ctr} \in \mathbb{R}^{4(N_n-1) \times N_n}
\]  

(4.49)

including Eq. (4.48) for each node in \(K_{ctr}\). Merging the complete grid model (4.40) with Eq. (4.44), (4.47) and (4.49), we obtain:

\[
\frac{d}{dt} \begin{bmatrix} z \\ x \\ \hat{U}_m \end{bmatrix} = \begin{bmatrix} A_n & B_nC_n & D_nK_{ctr} \\ B_nC_n & A_q & 0 \\ \Omega_rT_{ctr} & 0 & -\Omega_r \end{bmatrix} \begin{bmatrix} z \\ x \\ \hat{U}_m \end{bmatrix}
\]  

(4.50)

On the other hand, starting from the reduced order grid model (4.41) and from Eq. (4.44), (4.47) and (4.49), we obtain:

\[
\frac{d}{dt} \begin{bmatrix} z \\ \hat{U}_m \end{bmatrix} = \begin{bmatrix} A_n + B_nH_yC_n & D_nK_{ctr} \\ \Omega_rT_{ctr} & -\Omega_r \end{bmatrix} \begin{bmatrix} z \\ \hat{U}_m \end{bmatrix}
\]  

(4.51)

The feedback model that we have obtained is represented in Fig. 4.3 for the complete grid model (4.40). Once the state-space
model of the closed-loop system has been achieved, an eigenvalue analysis of the state matrix of Eq. (4.50) or (4.51) can be done to study the dynamics of the closed-loop system and to design properly the regulator.

4.4 Conclusions of chapter 4
In this chapter, an approach to address the small-signal stability analysis has been proposed: it allows both the study of the dynamics of the distribution grid as a whole and also the design of a secondary level control for the DERs. Indeed, we have analyzed first an open-loop configuration of the grid, that is when the DERs receive constant power references, and then a closed-loop configuration with the Q − V droop control for DERs. Particular detail has been given to the modeling choices for the PV DERs and the loads. This approach, compared to [57], is more detailed.

Electric grid model

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} z \\ \dot{x} \end{bmatrix} &= \begin{bmatrix} A_n & B_n C_g \\ B_g C_n & A_g \end{bmatrix} \begin{bmatrix} z \\ \dot{x} \end{bmatrix} + \begin{bmatrix} D_n \\ 0 \end{bmatrix} S \\
\dot{U} &= T_{ctr} \begin{bmatrix} \dot{z} \\ \dot{x} \end{bmatrix} \\
\frac{d\dot{U}_m}{dt} &= -\Omega_r \dot{U}_m + \omega_r \dot{U} \\
S &= K_{ctr} \dot{U}_m
\end{align*}
\]

Figure 4.3: Representation of the closed-loop, from the complete grid model (4.40) and the Q − V droop controller model, of Eq. (4.47) and (4.49) considering the load dynamics and the coupled power dependencies due to cables with low X/R ratio, while [57] neglects all these aspects, modeling only the PV DERs with their reactive power loops.

5. Stability analysis of A2A grid
In this chapter, some results obtained from the small-signal analysis of chapter 4 are shown together with some time-domain simulations. In particular, we will focus both on the open-loop system, that is the distribution grid with constant power references for the inverters, as in Sec. 4.1, and on the closed-loop system, referring to the local Q − V droop control, as in Sec. 4.2.

5.1 Test-case description
All the results of this chapter are based on the A2A grid benchmark that has been described in Sec. 3.3 and all its parameters are used here. Since the LV backbones are 4 wire cables and all the users and generators are single-phase and, the grid may be unbalanced. Thus, its analysis involves three independent analyses: one for each phase or with the three d_q0 components. For this reason, we choose to study only one phase because the others are similar, and among them, we choose to focus on phase
A1. For the dynamic study we consider the node model of Sec. 4.1.4 and so additional parameters have to be provided. To fix these parameters, these choices have been done:

- The nominal powers of loads and generators are set equal to the rated powers of Tab. 3.7.
- The PV units inject all the power ($P_{g,nom}$) at unity PF, during the open-loop operation.
- 40% of the load power ($S_{l,nom}$) is given by the linear RLC load, while the remaining 60% is from the dynamic constant power load (for all the nodes with a passive user).
- All the constant power loads have a PF (PFP load) of 0.85 (inductive), while the RLC loads have a 0.9 (inductive) PF (refer to the $R_{load}$, $L_{load}$ and $C_{load}$).
- The time constants of the PV generator models ($\tau_g$) are set randomly between 5 ms and 30 ms (they are different from one node to the other).
- The time constants of the load models ($\tau_l$) are set randomly between 1 ms and 5 s (they are different from one node to the other).

From these choices, the resulting parameters for the nodes are reported in Tab. 5.1: to see the correspondence between the node number and the node label, refer to Tab. 5.2.

**5.2 Open-loop dynamic results**

In this section, the eigenvalue analysis presented in Sec. 4.1 is used to study the dynamic characteristics of the grid in an open-loop operation.
loop configuration (no secondary controller that changes the power references). In Fig. 5.1, there are the steady-state voltage amplitudes of all nodes: the steady-state solution of the grid is necessary to perform the linearization of the mathematical model. The dynamic study of the grid is done in terms of eigenvalue analysis of the matrix $A_{tot}$ in Eq. (4.40) and (4.41). Both sets of eigenvalues have absolute values that range from factions of 1 up to 106, and for this reason we present them in complex planes after applying this compressing function:

$$f(v_i) = \begin{cases} 
    v_i & \text{if } |v_i| \leq 1 \\
    \frac{v_i}{1 + \log |v_i|} & \text{otherwise}
\end{cases}$$

This means that the complex planes we are going to show do not contain the eigenvalues $v_i$ as they are, but they show $f(v_i)$. This operation makes more readable the graphs themselves. Fig. 5.2 shows the eigenvalues of the $A_{tot}$ matrices of the models (4.40) and (4.41) to evaluate the dynamic characteristics of the grid, in terms of modes of the system: remember that among the states of the two models there are all the voltages of the grid. This figure also shows the approximations that are introduced by the static model of the grid, with Eq. (4.18). The eigenvalues are shown in the complex plane, by applying the function $f(\cdot)$, and also with the damping factors $\xi$ as function of the frequency of the eigenvalue, that is its absolute value divided by $2\pi$. We note the presence of modes with a very extended range of frequencies and that all of them have negative real parts, leading to a stable dynamic system. The slowest modes of the system are real eigenvalues, while some complex conjugate eigenvalues appear at frequencies near 100 Hz. Furthermore, from these graphs we can observe that the dynamic behavior of the two systems is the same for low frequency eigenvalues, while it is different for fast modes. With other words, the eigenvalues are almost the same for the two models, if they are near the origin of the complex plane or if they have small frequency, while they are different far from the origin or for high frequency. So we can conclude that both models, i.e. Eq. (4.40) and Eq. (4.41), describe in the same manner the important dynamic behaviors of the grid that are the slowest dynamics. Note that the first differences appear for eigenvalues with frequencies of hundreds of Hz.

### 5.3 Closed-loop dynamic results

In this section, we evaluate the dynamic behavior of the $Q-V$ droop control, considering only the complete model of the grid (4.50). We evaluate and compare the eigenvalues of the state matrix of system (4.50) with those of the open-loop model (4.40), to see how the dynamic characteristics of the grid are influenced by the secondary level voltage control. In the small-signal model, we assume that all the PV DERs work within the droop area and not within the dead-bands of the curve in Fig. 2.15, irrespective of the output voltage. This means that all the $k_v$ values in Eq. (4.48) are set to be different from zero, for the nodes that contain a DER. This choice has been done, because we agree that in this situation the dynamic behavior of the grid is most strongly affected, rather than in the situation where only few inverters are supporting the voltage via reactive power injection. Also the paper [57] states a similar concept. The results of Fig. 5.3 are for the closed-loop system with a bandwidth of the voltage amplitude measure $\omega_r = \omega_0$, Eq. (4.45), for all the inverters of the grid. Instead, the results of Fig. 5.4 are for a bandwidth $\omega_r = \omega_0/10$. The droop control moves slightly some poles with intermediate frequencies, near 10 Hz: these differences are more evident in Fig. 5.4, where a slow voltage measure causes the reduction of the damping factors of some eigenvalues (near 10 Hz frequency). However these variations are very slight.
5.4 Time-domain results

This section shows some time-domain simulations of the small-signal model and of the initial non-linear model (before the linearization). The small-signal model that has been used for the simulation is in Eq. (4.40), while the non-linear models used here, one for the open-loop system and another for the closed-loop system, consider the non-linearity that is introduced by the constant power loads and generators, see Sec. 4.1.4. Moreover, the closed-loop non-linear system considers also the non-linearity of the local droop control, this means that in the time-domain simulations we are considering the exact $Q - V$ drop of Fig. 2.15. For this system we consider only the bandwidth of the voltage measure $\omega_r = \omega_0$ for all the DERs.

5.4.1 Power reference variation

Here, the simulations of the small-signal model (4.40), the open-loop and closed-loop non-linear systems show the response of the grid to a change of the active power reference of the inverter at node C35: its active power reference is suddenly decreased of 1 kW at $t = 3$ s. The results are in terms of voltage amplitude deviation (Fig. 5.5) and voltage phase deviation (Fig. 5.6) from the steady-state solution for the node C29.

Figure 5.2: Comparison between the eigenvalues of the matrix $A_{tot}$ of Eq. (4.40) with blue dots and of Eq. (4.41) with red circles: on the left, on the complex plane (applying the function $f(\cdot)$ of Eq. (5.1)) and on the right, with damping factor as function of frequency; the dotted circle has radius 1 and center on the axis origin.
Figure 5.3: Comparison between the eigenvalues of the open-loop system, i.e. of the state matrix in Eq. (4.40), with blue dots, and of the closed-loop system, that is Eq. (4.50), with red circles: on the left, on the complex plane (applying the function $f(\cdot)$ of Eq. (5.1)) and on the right, with damping factor as function of frequency; these results are for a bandwidth for the voltage amplitude measure of $\omega = \omega_0$ for all the inverters; the dotted circle has radius 1 and center on the axis origin.

Figure 5.4: Comparison between the eigenvalues of the open-loop system, i.e. of the state matrix in Eq. (4.40), with blue dots, and of the closed-loop system, that is Eq. (4.50), with red circles: on the left, on the complex plane (applying the function $f(\cdot)$ of Eq. (5.1)) and on the right, with damping factor as function of frequency; these results are for a bandwidth for the voltage amplitude measure of $\omega = \omega_0 / 10$ for all the inverters; the dotted circle has radius 1 and center on
the axis origin

Figure 5.5: Voltage amplitude deviation from the steady-state solution for the node C29 after a step reduction of 1 kW of the active power reference of the inverter at node C35: comparison among the small-signal open-loop model (4.40) and the open-loop and closed-loop non-linear models

Figure 5.6: Voltage phase deviation from the steady-state solution for the node C29 after a step reduction of 1kW of the active power reference of the inverter at node C35: comparison among the small-signal open-loop model (4.40) and the open-loop and closed-loop non-linear models

The active and reactive powers injected by the PCC are shown in Fig. 5.7. From these figures, we can see that the dynamic model describes both very slow dynamic (orders of tens of seconds) and fast dynamics (orders of milliseconds).

In Fig. 5.5 and Fig. 5.6, the results of the small-signal open-loop model and the results for the non-linear open-loop model are almost overlapped, meaning that the small-signal model describes quite well the non-linear system. It is interesting to note from Fig. 5.5, that the droop control can limit the steady-state voltage variation at the node C29 for the considered transient, but its effect is limited. However, we note that the reactive power injection has more influence on the voltage phase rather than on the voltage amplitude (compare Fig. 5.5 and 5.6), because the X/R ratio of the cables is quite low. From the Fig. 5.7, we can note that injecting reactive power with the DERs (with the droop control) can keep the PF of the PCC closer to 1, as anticipated in Sec. 2.6.

5.4.2 PCC voltage variation

Here, the simulations of the open-loop and closed-loop non-linear systems show the response of the grid to a drop of the PCC
voltage amplitude of 0.02 p.u. at t = 3 s. The results are in terms of voltage amplitude deviation (Fig. 5.8) and voltage phase deviation (Fig. 5.9) from the steady-state solution for the node C29. Also the active and reactive powers injected by the PCC are shown in Fig. 5.10. Also in this simulation, we can see from Fig. 5.8, that the droop control can limit the voltage variation at the node C29 for the considered transient. However, again we note that the reactive power injection has more influence on the voltage phase rather than on the voltage amplitude (compare Fig. 5.8 and 5.9), because of the cables with low X/R. Also the observation of the increased PF at the PCC is still valid: from the Fig. 5.10, we see that injecting reactive power with the DERs can keep the PF of the PCC closer to 1, without requiring excessive reactive power from the PCC. From all these time-domain simulations we can see that there are different types of dynamic within this grid models and they are quite separated in term of frequency: there are the faster dynamics of the cables with sub-millisecond time responses, there are the dynamics of the inverters an their secondary controllers with times scale of tens of millisecond or hundreds of millisecond and, finally, there are the load dynamics that can last for hundreds of millisecond up to tens of second. Our dynamic models can represent all of them, both for the analytic study with the eigenvalues and also for the time-domain simulations.

Figure 5.6: Voltage phase deviation from the steady-state solution for the node C29 after a step reduction of 1 kW of the active power reference of the inverter at node C35: comparison among the small-signal open-loop model (4.40) and the open-loop and closed-loop non-linear models

Figure 5.7: Active and reactive powers injected at the PCC after a step reduction of 1 kW of the active power reference of the inverter at node C35: comparison among the small-signal open-loop model (4.40) and the open-loop and closed-loop non-linear models
Figure 5.8: Voltage amplitude deviation from the steady-state solution for the node C29 after a step reduction of the PCC voltage amplitude of 0.02 p.u.: comparison between the open-loop and closed-loop non-linear models

Figure 5.9: Voltage phase deviation from the steady-state solution for the node C29 after a step reduction of the PCC voltage amplitude of 0.02 p.u.: comparison between the open-loop and closed-loop non-linear models
Figure 5.10: Active and reactive powers injected at the PCC after a step reduction of the PCC voltage amplitude of 0.02 p.u.: comparison between the open-loop and closed-loop non-linear models

References


